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Coordination Failure in Foreign Aid*

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Abstract

This paper analyzes the allocation of foreign aid to various sectors in a recipient developing country. Donors tend to favor social sectors over other public expenditure programs. Due to incomplete information, donors may concentrate too much on priority sectors, leaving lower-priority yet important sectors lacking funds. Alternatively there may be gaps in services in priority areas because of the information problem. The more similar preferences the donors have, the more scope there is for coordination failure. Therefore improving information is particularly important when the parties have similar priorities. A joint database on planned projects and budget allocations in each recipient country would provide such information. The point of the paper is that such databases should have both information on current projects and forward-looking information on the planned activities needed to improve aid coordination. It also analyzes the aid fungibility problem in an incomplete information setting, and finds that incomplete information reduces the fungibility problem. On the other hand, incomplete information introduces coordination failure and the allocation can be inferior for *both* the recipient *and* the donor.

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1 Introduction

Recent evidence demonstrates that aid is effective in raising economic growth and reducing income poverty in "good" environments but ineffective in "poor" environments; the result holds both when the environment is defined narrowly as fiscal, monetary, and trade policies, and more broadly in terms of a wide range of policies and public institutions (Burnside and Dollar 2000; and Collier and Dollar 2002). This evidence also shows that the policy and institutional environment of the recipient country has only a limited impact on the present allocation of aid. Because donors have multiple objectives for their aid, such as strategic and historical considerations,[1] the actual allocation of aid differs significantly from, for example, the poverty-efficient allocation that would target poor countries with good policies and institutions. According to one calculation, the present allocation of aid lifts 10 million people out of poverty annually, while its poverty-efficient allocation would double the effect of aid on growth and poverty (Collier and Dollar 2002). Hence, in line with this evidence, aid effectiveness could be much enhanced if donors improved their allocation of aid between recipient countries toward those that are poor and have good policies and institutions.

This paper takes the aid effectiveness debate beyond country allocations and contributes to a theoretically underexplored area. It argues that, once aid has been allocated to a poor recipient country with good policies, overcoming the coordination failure in aid can further enhance its effectiveness. Donors allocate their funds between regions, sectors and activities in a recipient country. When there is incomplete information about the other donors' aid, coordination failure can occur. The resulting allocation can be such that all donors regret it ex post. Donors tend to favor social sectors (health and education) over other public expenditure programs (e.g., transportation). Due to incomplete information, they may all concentrate too much on the priority sectors, leaving the lower-priority yet important sectors lacking funds. Alternatively there may be gaps in services in the priority areas because of the information problem.[2]

The information problem often extends to the recipient countries as well; many recipients are not aware of donor activities in any detail ex ante (or even

[1] Alesina and Dollar (2000).

[2] See Mackinnon (2003) for an argument of misallocation of donations that does not depend on information problems. Donors underfund activities they agree about and overfund each donor's idiosyncratic schemes.

ex post). For example, in Vietnam 25 official bilateral donors, 19 official multilateral donors and about 350 international NGOs (non-governmental organizations) were operating in 2002. They accounted for over 8,000 projects.[3] With such a great number of projects, it is difficult to keep track of the planned (or even the past) activities at the aggregate level. This is true for the recipient government and for the donors. For the donors the information problem is exacerbated by the fact that they operate in numerous developing countries. For example, bilateral donors assist on average 107 recipient countries each.[4] Fragmentation imposes high transaction costs on the recipient countries, not least because the donors have different reporting requirements. Great emphasis has recently been placed on harmonizing donor practices to reduce transaction costs. This paper points to a less discussed problem arising from fragmentation. Fragmentation makes acquiring information about other donors' planned activities and budget allocations complex. This incomplete information can lead to inefficient allocation decisions and coordination failure among donors.

There are often genuine differences between donors and recipients about how to allocate public funds in the recipient developing country. This leads to the well-known aid fungibility problem. The donors aim to increase funding in their priority sectors but the government responds to aid by shifting funding away from those sectors. Therefore aid in effect finances indirectly some other, potentially unproductive activities. The paper analyzes the aid fungibility problem in an incomplete information setting, which to my knowledge is novel. The analysis finds that incomplete information reduces the fungibility problem because the recipient government does not have the information to fully respond to donors' decisions. On the other hand, incomplete information introduces coordination failure and the allocation can be inferior to *both* the recipient *and* the donor.

The analysis of three versions of the allocation game with increasingly different preferences finds that the more similar the preferences are, the more scope there is for coordination failure. When similar agents pursue the common good, misallocation due to incomplete information can be considerable. When the agents have very different priorities about their activities in the recipient developing country, information problems are minimal.

Information problems arise because donors run their own projects in the

[3] Acharya, Fuzzo de Lima, and Moore (2003).
[4] Acharya, Fuzzo de Lima, and Moore (2003).

recipient developing countries. One way of overcoming the information problem (and many other inefficiencies arising from the project approach[5]) is for donors to pool their funds within a sector in a recipient developing country. Alternatively the donors can channel the aid as budget support to the recipient country so that the government makes the allocation decisions. There are, however, strong motives for the donor agencies to maintain the project approach, e.g., to demonstrate visible results to the taxpayers in the donor country and to accomplish the type of projects the donor country prefers. Another way to overcome the coordination failure is to simply improve information about the donors' planned activities and budgets. According to the paper's results, improving information is particularly important when the parties have similar priorities. A joint database on planned projects and budget allocations in each recipient country would provide such information. Such databases should not have just information on the current projects, but also forward-looking information on the planned activities to improve aid coordination.

The rest of the paper is organized as follows. Section 2 sets up the model of two donors and a passive recipient. Section 3 analyzes coordination failure among donors. Section 4 introduces an active recipient with different preferences from the donor and analyzes how coordination failure can add to the aid fungibility problem. Section 5 concludes.

The literature on voluntary provision of public goods and the free-rider problem is related. (e.g., Olson 1965; Chamberlin 1974, 1976; McGuire 1974). See Kemp (1984) on an application to foreign aid. In this literature the level of contributions is endogenous, while the focus here is on allocating a fixed budget to a two-dimensional public good.

The structure of the model has similarities with networks, in particular Farrell and Saloner (1985). There are network externalities when the utility a consumer derives from a product depends on the number of other consumers with the same product. Consumers have a fixed budget (as in our model) and choose the product from two possible ones. There is incomplete information about the other consumers' preferences and coordination failure can occur: in equilibrium the consumers can choose an inferior good. In this paper's framework, there are network externalities as the donor's utility depends on the number of projects funded in each sector by the other donor. However, the incomplete information is not about the other donor's preferences but

[5] For other inefficiencies, see Kanbur, Sandler, and Morrison (1999).

about his budget size. Therefore the mechanics of the model and the insights arising from it are different.

Finally, there is an empirical literature on foreign aid fungibility (for a review, see Devarajan and Swaroop 1998). These studies analyze whether foreign aid for specific categories of expenditure is shifted by the recipient government. If the recipient government shifts funding as a response to foreign aid, aid is in effect financing indirectly some other, potentially unproductive activities. This paper analyzes the aid fungibility problem in an incomplete information setting, which to my knowledge is novel.

2 The model

Two donors, 1 and 2, operate in a recipient developing country in two sectors/activities/regions, 1 and 2. The following refers to sectors. The donors' objective is to increase welfare in the recipient developing country.

Each donor has a fixed budget. The focus is on the question how the donors allocate the budget between the sectors. This is the practical problem the donor agencies face. Discrete projects are analyzed assuming that donor i can finance n_i projects where $n_i \in \{0,1,2,3\}$. Most of the foreign aid is channelled as project aid. The size of the budget is private information to the donor. Donor i attaches prior probability p_n for donor j's budget to equal n. $\sum_{n=0}^{3} p_n = 1$.

The donors' utility depends on the total number of projects financed in each sector, $u\left(n_1^1+n_2^1, n_1^2+n_2^2\right)$, where n_i^j is the number of projects donor i finances in sector j. In other words, the donors are providing public goods. The donors have the same preferences. In Section 4 we analyze agents with different preferences. The utility is strictly increasing in the number of projects implying that the donors always exhaust their budget.

Assumption 1. (i) $u(\alpha+1,\beta) > u(\alpha,\beta)$ and

$$(ii)\ u(\alpha,\beta+1) > u(\alpha,\beta).$$

The donors view sector 1 as contributing more to welfare than sector 2. Donors tend to favor social sectors (health or education) over other public expenditure programs (e.g., transportation). For example five out of the eight Millennium Development Goals – agreed by the international donor community to reduce poverty – concern health and education.[6]

We define $\Delta u(\alpha, \beta) \equiv u(\alpha+1, \beta-1) - u(\alpha, \beta)$. $\Delta u(\alpha, \beta)$ gives the change in utility when one project is reallocated to the priority sector 1 given the original allocation (α, β). Assumption 2 signs $\Delta u(\alpha, \beta)$.

Assumption 2. (i) $\Delta u(\alpha, \beta) > 0$ if and only if $\alpha \leq \beta$.

(ii) $\Delta u(\alpha, \beta) < 0$ if and only if $\alpha > \beta$.

Property (i) means that the donors prefer to allocate more projects toward sector 1 while property (ii) states that the sectors are complementary and too much inequality in the resources is welfare reducing. These two properties order all reallocations by one project. The frequently used terms are given below.

$$\Delta u(0,1) > 0,\ \Delta u(1,1) > 0,\ \Delta u(2,2) > 0,\ \Delta u(3,3) > 0$$

$$\Delta u(2,1) < 0,\ \Delta u(3,2) < 0,\ \Delta u(4,2) < 0$$

We assume that the preferences are common knowledge. In reality allocation of foreign aid is not a one-shot game (although at the first instance we model it as such) but the same donors operate in the recipient countries over the years. The type of aid each donor prefers to give is revealed over time while their budgets vary each year. Therefore we have incomplete information about the budgets, not about the preferences. [7]

The recipient country is a passive player in this version of the model. The recipient does not have any funds or any bargaining power on how to allocate the donors' funds. In Section 4 we extend the analysis to an active recipient who has different preferences from the donor.

[6]The Millennium Development Goals are: (1) Eradicate extreme poverty and hunger. (2) Achieve universal primary education. (3) Promote gender equality and empower women. (4) Reduce child mortality. (5) Improve maternal health. (6) Combat HIV/AIDS, malaria, and other diseases. (7) Ensure environmental sustainability. (8) Develop a global partnership for development.

[7]One interpretation of the model is that there is incomplete information about the number of other donors operating in the recipient country in a particular year. But strictly speaking we are analyzing two donors.

3 Coordination failure among donors

3.1 Complete information

We first analyze the benchmark case of complete information. Each donor knows not only his own budget but also that of the other donor, that is they know what is the aggregate budget. Assumption 2 gives the optimal allocation of the aggregate budget which is:

(1,0) if and only if $n_1 + n_2 = 1$
(2,0) if and only if $n_1 + n_2 = 2$
(2,1) if and only if $n_1 + n_2 = 3$
(3,1) if and only if $n_1 + n_2 = 4$
(3,2) if and only if $n_1 + n_2 = 5$
(4,2) if and only if $n_1 + n_2 = 6$

Suppose $n_1 = 1$ and $n_2 = 2$. Table 1 reports possible allocation strategies for each donor and the resulting aggregate allocation. Donor 1's strategy is given vertically and donor 2's strategy horizontally.

Table 1. Aggregate Allocation

	$(2,0)$	$(1,1)$	$(0,2)$
$(1,0)$	$(3,0)$	$(2,1)^*$	$(1,2)$
$(0,1)$	$(2,1)^*$	$(1,2)$	$(0,3)$

There are two Nash equilibria: (i) donor 1 funds one project in sector 1 and donor 2 funds one project in each sector and (ii) donor 1 invests one project in sector 2 and donor 2 invests all the funds in sector 1. Both Nash equilibria lead to the same aggregate allocation (2,1). This allocation is efficient. This is true for any allocation game. The aggregate allocation is efficient and for any $n_1 + n_2 > 2$ there are multiple Nash equilibria to achieve that allocation. The donors are contributing toward public goods over which they have similar preferences. That is why the Nash equilibrium allocation with complete information is efficient.

Table 2 shows the equilibrium allocation. Donor 1's budget is vertical and donor 2's budget is horizontal.

Table 2. Complete Information

	0	1	2	3
0	(0,0)	(1,0)	(2,0)	(2,1)
1	(1,0)	(2,0)	(2,1)	(3,1)
2	(2,0)	(2,1)	(3,1)	(3,2)
3	(2,1)	(3,1)	(3,2)	(4,2)

The outcome is naturally biased toward sector 1 since both donors view it as more important.

3.2 Incomplete information

Then we turn to incomplete information. The donors know their own budget but not the size of the other donor's budget. They only know the probability distribution of the other donor's budget. It is the incomplete information about the budget size that leads to coordination failure. Coordination failure means that the equilibrium allocation is Pareto dominated; the resulting allocation is such that both donors regret it ex post. The coordination failure can be of two types. First, the less important sector 2 does not get enough resources as both donors concentrate too much on priority sector 1. Second, in an attempt at a more balanced allocation, the donors invest too much in sector 2 and priority sector 1 does not get enough resources.

We describe the allocation decisions as sharing strategies, sharing pointing to how much resources are allocated to the lower priority sector 2. A sharing strategy $s_i = (\alpha, \beta, \gamma)$ denotes that given budget size 1 the donor allocates α to sector 2, from budget size 2 β and from budget size 3 γ projects are funded in sector 2. E.g. $s_i = (0,0,0)$ means that donor i invests all the funds in sector 1 whatever the budget and $s_i = (1,2,3)$ denotes that the donor allocates all the funds in sector 2.[8]

In what follows we find the various types of equilibria that can exist in this allocation game. After examining each equilibrium in detail we analyze the general pattern that emerges from them.

We start by examining no sharing strategies.

Proposition 1 *It is not a Bayesian equilibrium for both donors to invest all their budget in sector 1.*

Proof. Suppose that donor 2's equilibrium sharing strategy is $s_2^* = (0,0,0)$, i.e. donor 2 invests the whole budget in sector 1. Given s_2^* is donor

[8] From the complete information case we know that the donors never wish to have more than 2 projects in sector 2. Therefore $s_i = (1,2,3)$ cannot be an equilibrium strategy.

1 better off with strategy $s_1 = (0, 0, 1)$ or with $s_1 = (0, 0, 0)$? These strategies differ only for $n_1 = 3$. To find which strategy gives donor 1 a higher utility we only need to check the outcome for the large budget 3. Donor 1 is better off with $s_1 = (0, 0, 1)$ if and only if:

$$p_0 u\left(2,1\right) + p_1 u\left(3,1\right) + p_2 u\left(4,1\right) + p_3 u\left(5,1\right) \tag{1}$$

$$> p_0 u\left(3,0\right) + p_1 u\left(4,0\right) + p_2 u\left(5,0\right) + p_3 u\left(6,0\right)$$

Equation(1) simplifies to:

$$p_0 \Delta u(2,1) + p_1 \Delta u(3,1) + p_2 \Delta u(4,1) + p_3 \Delta u(5,1) < 0 \tag{2}$$

Equation (2) clearly holds since each term is negative by Assumption 2. Therefore it is not a Bayesian equilibrium for both donors to choose $s_i = (0, 0, 0)$. Q.E.D.

Proposition 1 shows that if donor i invests all the budget in the priority sector 1, i.e. chooses $s_i = (0, 0, 0)$, it is not optimal for donor j to do the same. If donor j also invests all the budget in sector 1 the resulting allocation would be too unequal. Therefore it is not a Bayesian equilibrium for both donors to choose $s_i = (0, 0, 0)$.

Now that we have established that there is sharing in equilibrium we proceed to ask how much. In the absence of donor j, donor i would focus on the more important sector 1 and only invest one project in sector 2 if the budget is 3. There is an equilibrium where both donors follow this "monopoly" strategy.

Proposition 2 *A symmetric Bayesian equilibrium $s_i^* = (0, 0, 1)$ for $i = 1, 2$ exists if*

(i) $p_0 \underset{+}{\Delta u}(0,1) + p_1 \underset{+}{\Delta u}\left(1,1\right) + p_2 \underset{-}{\Delta u}\left(2,1\right) + p_3 \underset{+}{\Delta u}\left(2,2\right) > 0$

(ii) $p_0 \underset{+}{\Delta u}(1,1) + p_1 \underset{-}{\Delta u}\left(2,1\right) + p_2 \underset{-}{\Delta u}\left(3,1\right) + p_3 \underset{-}{\Delta u}\left(3,2\right) > 0$ *and*

(iii) $p_0 \underset{+}{\Delta u}(1,2) + p_1 \underset{+}{\Delta u}\left(2,2\right) + p_2 \underset{-}{\Delta u}\left(3,2\right) + p_3 \underset{+}{\Delta u}\left(3,3\right) > 0.$

Proof. See the Appendix.

If the conditions in Proposition 2 are satisfied[9] both donors choose $s_i^* = (0,0,1)$. We denote the sign of each term under it to ease reading. Condition (ii) is the most stringent; only the first term has the same sign as the whole equation. This condition is for the donor with budget size 2 to prefer investing it all in sector 1. Given $s_j = (0,0,1)$ and $n_i = 2$ donor i wishes to invest all the funds in sector 1 if and only if there is a high probability that $n_j = 0$. For $n_j > 0$ this strategy would result in overfunding of sector 1.

Corollary 1 gives an intuitive sufficient condition for all the conditions in Proposition 2 to be satisfied.

Corollary 1 *A symmetric Bayesian equilibrium* $s_i^* = (0,0,1)$ *for* $i = 1,2$ *exists if* $p_0 \to 1$.

Proof. Straightforward given Proposition 2.

When it is very likely that the other donor has a zero budget each donor behaves as if he were the only donor active in the recipient country and follows the "monopoly" strategy. The resulting allocation is given in Table 3.

Table 3. Symmetric minimal sharing equilibrium: $s_1 = (0,0,1)$, $s_2 = (0,0,1)$

	0	1	2	3
0	(0,0)	(1,0)	(2,0)	(2,1)
1	(1,0)	(2,0)	(3,0)*	(3,1)
2	(2,0)	(3,0)*	(4,0)*	(4,1)*
3	(2,1)	(3,1)	(4,1)*	(4,2)

In this equilibrium there is too little sharing. There is coordination failure in five eventualities marked by * . Ex post both donors regret that too much of the aggregate funds are invested in the priority sector 1. The coordination failure occurs for budget size 2.

To solve the above coordination failure, the donors could share also from budget size 2. Proposition 3 establishes when this is a Bayesian equilibrium.

[9] To derive both necessary and sufficient conditions we would need to consider also reallocations by two projects and we would need more structure to the utility functions than we currently have. E.g. we would need to rank $u(2,2)$ and $u(4,0)$. We have chosen not to proceed this way and therefore most of the propositions give sufficient conditions.

Proposition 3 *A symmetric Bayesian equilibrium* $s_i^* = (0,1,1)$ *for* $i = 1,2$ *exists if and only if*

$$p_0 \Delta \underset{+}{u(1,1)} + p_1 \Delta u \underset{-}{(2,1)} + p_2 \Delta u \underset{+}{(2,2)} + p_3 \Delta u \underset{-}{(3,2)} < 0$$

Proof. See the Appendix.

The condition in Proposition 3 is for the donor preferring to share a budget of 2 equally between the two sectors. Given $n_i = 2$ and $s_j = (0,1,1)$ donor i prefers sharing if $(p_1 + p_3)$ is large enough. For $n_j = 1$ donor j invests one project in sector 1. Additional two projects would lead to too unequal allocation. Thus donor i prefers sharing. With $n_j = 3$ donor j invests two projects in sector 1 and again additional two projects by donor i would mean that sector 1 is overfunded.

Corollary 2 gives sufficient conditions for this symmetric equilibrium to exist.

Corollary 2 *A symmetric Bayesian equilibrium* $s_i^* = (0,1,1)$ *for* $i = 1,2$ *exists if (i)* $p_1 \to 1$ *or (ii)* $p_3 \to 1$.

Proof. Straightforward given Proposition 3.

The other donor is likely to have a larger budget than in the previous corollary. Therefore the expected aggregate budget is higher and more of it should go to sector 2. This can be achieved by sharing strategy $s_i^* = (0,1,1)$. The allocation of funds is given in Table 4.

Table 4. Symmetric sharing equilibrium: $s_1 = (0,1,1)$, $s_2 = (0,1,1)$

	0	1	2	3
0	(0,0)	(1,0)	(1,1)**	(2,1)
1	(1,0)	(2,0)	(2,1)	(3,1)
2	(1,1)**	(2,1)	(2,2)**	(3,2)
3	(2,1)	(3,1)	(3,2)	(4,2)

This time we have oversharing in equilibrium in three eventualities marked by **. Both sectors are funded equally in the three eventualities while the donors would get higher utility from reallocating one project to the priority sector. The coordination failure occurs again for budget size 2.

To balance the two types of coordination failure (undersharing and oversharing) donor i could follow $s_i = (0,0,1)$ and donor j could choose $s_j =$

$(0,1,1)$. The following proposition establishes the sufficient conditions for this to be a Bayesian equilibrium.

Proposition 4 *An asymmetric Bayesian equilibrium* $s_i^* = (0,0,1)$ *and* $s_j^* = (0,1,1)$ *exists if*

(i) $-p_2 \underset{+}{\Delta u}(2,2) < p_0 \underset{+}{\Delta u}(1,1) + p_1 \underset{-}{\Delta u}(2,1) + p_3 \underset{-}{\Delta u}(3,2) < -p_2 \underset{-}{\Delta u}(3,1)$

(ii) $p_0 \underset{+}{\Delta u}(0,1) + p_1 \underset{+}{\Delta u}(1,1) + p_2 \underset{-}{\Delta u}(2,1) + p_3 \underset{+}{\Delta u}(2,2) > 0$ *and*

(iii) $p_0 \underset{+}{\Delta u}(1,2) + p_1 \underset{+}{\Delta u}(2,2) + p_2 \underset{-}{\Delta u}(3,2) + p_3 \underset{+}{\Delta u}(3,3) > 0.$

Proof. See the Appendix.

None of the conditions in Proposition 4 is particularly stringent. But for this equilibrium we cannot find a simple probability distribution that would satisfy all the conditions. Uniform distribution with some additional restrictions fulfills the conditions as the following corollary shows.

Corollary 3 *An asymmetric Bayesian equilibrium* $s_i^* = (0,0,1)$ *and* $s_j^* = (0,1,1)$ *exists if*

(i) $p_0 = p_1 = p_2 = p_3$,

(ii) $-\underset{+}{\Delta u}(2,2) < \underset{+}{\Delta u}(1,1) + \underset{-}{\Delta u}(2,1) + \underset{-}{\Delta u}(3,2) < -\underset{-}{\Delta u}(3,1)$ *and*

(iii) $\underset{+}{\Delta u}(1,2) + \underset{+}{\Delta u}(2,2) + \underset{+}{\Delta u}(3,3) + \underset{-}{\Delta u}(3,2) > 0.$

Proof. See the Appendix.

When all the budget sizes are equally likely it is optimal to balance oversharing and undersharing. The resulting equilibrium is in Table 5.

Table 5. Asymmetric sharing equilibrium: $s_1 = (0,0,1)$, $s_2 = (0,1,1)$

	0	1	2	3
0	(0,0)	(1,0)	(1,1)**	(2,1)
1	(1,0)	(2,0)	(2,1)	(3,1)
2	(2,0)	(3,0)*	(3,1)	(4,1)*
3	(2,1)	(3,1)	(3,2)	(4,2)

Asymmetric sharing strategies eliminate two out of the three oversharing eventualities of Table 4 and three out of the five undersharing eventualities of Table 3.

Alternatively the donors can specialize in different sectors. With specialization donor i invests all his budget in sector 1 and donor j specializes in sector 2. Donor j does not fund more than two projects in sector 2 as we know from the complete information case that the donors never wish to have more than two projects in sector 2. Proposition 5 shows when specialization is a Bayesian equilibrium.

Proposition 5 *Specialization* $s_i^* = (0,0,0)$ *and* $s_j^* = (1,2,2)$ *is a Bayesian equilibrium if*

(i) $p_0 \underset{-}{\Delta u}(2,1) + p_1 \underset{+}{\Delta u}(2,2) + p_2 \underset{+}{\Delta u}(2,3) + p_3 \underset{+}{\Delta u}(3,3) > 0,$

(ii) $p_0 \underset{+}{\Delta u}(0,1) + p_1 \underset{+}{\Delta u}(1,1) + p_2 \underset{-}{\Delta u}(2,1) + p_3 \underset{-}{\Delta u}(3,1) < 0,$

(iii) $p_0 \underset{+}{\Delta u}(1,1) + p_1 \underset{-}{\Delta u}(2,1) + p_2 \underset{-}{\Delta u}(3,1) + p_3 \underset{-}{\Delta u}(4,1) < 0,$

(iv) $p_0 \underset{+}{\Delta u}(0,2) + p_1 \underset{+}{\Delta u}(1,2) + p_2 \underset{+}{\Delta u}(2,2) + p_3 \underset{-}{\Delta u}(3,2) < 0$ *and*

(v) $p_0 \underset{+}{\Delta u}(1,2) + p_1 \underset{+}{\Delta u}(2,2) + p_2 \underset{-}{\Delta u}(3,2) + p_3 \underset{-}{\Delta u}(4,2) < 0.$

Proof. See the Appendix.

Out of the five conditions in Proposition 5 only (i) is for donor i. Given donor j is specializing in the low priority sector it is obvious that donor i has very strong incentives to specialize in sector 1 whatever the probability distribution of donor j's budget. The only case when donor i would not allocate all the funds in sector 1 is when $n_i = 3$ and donor j is very likely to have a zero budget. Then donor i would fund one project in sector 2. But if p_0 is low enough $s_i = (0,0,0)$ is best response to $s_j = (1,2,2)$ as condition (i) requires.

It is clearly more difficult for a donor to specialize in the lower priority sector. Conditions $(ii) - (v)$ in Proposition 5 have to be fulfilled for specialization in sector 2 to be optimal. The most stringent of them is condition (iv). This condition is for donor j to prefer to invest all his budget $n_j = 2$ in sector 2 given $s_i = (0,0,0)$. Donor j gives higher priority to sector 1 and only wishes to invest two projects in sector 2 if sector 1 receives more than that from donor i. This is the case when donor i is very likely to have a budget of 3.

Corollary 4 shows that not only condition (iv) but all the conditions in Proposition 5 are satisfied for p_3 large.

Corollary 4 *Specialization equilibrium $s_i^* = (0,0,0)$ and $s_j^* = (1,2,2)$ exists if $p_3 \rightarrow 1$.*

Proof. Straightforward given Proposition 5.

Why would a donor ever specialize in the low priority sector? When he knows that the other donor is funding the priority sector and has large enough budget to be able to deal with that on his own, then he is happy to concentrate on the less important sector. This is the case when p_3 is large. Furthermore, when the expected aggregate budget is large (when p_3 is large) the lower priority sector 2 should be relatively well funded and accordingly one donor specializing in sector 2 is optimal. Allocation with specialization is demonstrated in Table 6.

Table 6. Specialization equilibrium: $s_1 = (0,0,0)$, $s_2 = (1,2,2)$

	0	1	2	3
0	(0,0)	(0,1)**	(0,2)**	(1,2)**
1	(1,0)	(1,1)**	(1,2)**	(2,2)**
2	(2,0)	(2,1)	(2,2)**	(3,2)
3	(3,0)*	(3,1)	(3,2)	(4,2)

There is oversharing in seven eventualities and in four of them the lower priority sector 2 is actually better funded than sector 1. Additionally there is one eventuality of undersharing. If the donor specializing in the priority sector has a low budget, specialization leads to oversharing.

Much of the coordination failure in Table 5 occurs when the donor specializing in sector 2 has a budget of 2. If that donor specializes less in sector 2 and chooses $s_j = (1,1,2)$ some of this coordination failure is eliminated. In equilibrium this strategy is chosen in combination with $s_i = (0,0,1)$: now also donor i specializes only partially in sector 1.

Proposition 6 *Partial specialization* $s_i^* = (0,0,1)$ *and* $s_j^* = (1,1,2)$ *is a Bayesian equilibrium if*

(i) $p_0 \underset{+}{\Delta u}(0,1) + p_1 \underset{+}{\Delta u}(1,1) + p_2 \underset{-}{\Delta u}(2,1) + p_3 \underset{+}{\Delta u}(2,2) < 0,$

(ii) $p_0 \underset{+}{\Delta u}(1,1) + p_1 \underset{-}{\Delta u}(2,1) + p_2 \underset{-}{\Delta u}(3,1) + p_3 \underset{-}{\Delta u}(3,2) < 0$ *and*

(iii) $p_0 \underset{+}{\Delta u}(1,2) + p_1 \underset{+}{\Delta u}(2,2) + p_2 \underset{-}{\Delta u}(3,2) + p_3 \underset{+}{\Delta u}(3,3) < 0.$

Proof. See the Appendix.

Conditions (i) and (iii) in Proposition 6 are stringent. Condition (i) is for donor j with $n_j = 1$ to prefer investing in sector 2 given $s_i = (0,0,1)$. If p_2 is large, donor i is likely to invest two projects in sector 1. Then donor j is better off funding one project in the lower priority sector 2 or else sector 1 is overfunded. Condition (iii) is for donor j with $n_j = 3$ to prefer funding two projects in sector 2. If p_2 is large, donor i is likely to invest two projects in the priority sector 1. When donor j funds one project in sector 1 and two projects in sector 2, the aggregate allocation is in favor of the priority sector 1 but not too unbalanced.

It turns out that also condition (ii) is satisfied for p_2 large as Corollary 5 proves.

Corollary 5 *Partial specialization with* $s_i^* = (0,0,1)$ *and* $s_j^* = (1,1,2)$ *is a Bayesian equilibrium if* $p_2 \to 1$.

Proof. Straightforward given Proposition 6.

The other donor is likely to have a fairly large budget and therefore nearly be able to deal with one sector. Specialization is not complete: for large budget the donors allocate some funds to the other sector. Table 7 shows the resulting allocation.

Table 7. Partial specialization equilibrium: $s_1 = (0,0,1)$, $s_2 = (1,1,2)$

	0	1	2	3
0	(0,0)	(0,1)**	(1,1)**	(1,2)**
1	(1,0)	(1,1)**	(2,1)	(2,2)**
2	(2,0)	(2,1)	(3,1)	(3,2)
3	(2,1)	(2,2)**	(3,2)	(3,3)**

Compared to specialization equilibrium partial specialization eliminates two out of the seven oversharing eventualities and reduces coordination failure in one eventuality. The one case of undersharing is eliminated as well. On the other hand, two new cases of oversharing occur.

The five equilibria analyzed above are the only pure strategy Bayesian equilibria that can exist in this allocation game. We have restricted analysis to weakly increasing sharing strategies, i.e. $s_i = (\alpha, \beta, \gamma)$ such that $\alpha \leq \beta \leq \gamma$.

The pattern that emerges from these results is that:

- When the other donor is likely to have a small budget (0 or 1) or the distribution is uniform both donors concentrate on the priority sector and share marginally.
- When the other donor is likely to have a large budget (2 or 3) the donors specialize.[10]

If it is very likely that the other donor has a large budget, the donors specialize. From large aggregate budget quite a lot should go to the lower priority sector 2 making it feasible for one donor to specialize in the lower priority sector. With a large budget the donor is able to deal with one sector on his own.

If it is very likely that the other donor has a small budget, each donor concentrates on the priority sector and invests a little in sector 2. From small aggregate budget most funds should go to the priority sector 1. This is achieved by both donors focusing on the priority sector.

[10] For $p_3 \to 1$ two equilibria exist: specialization and symmetric sharing equilibrium.

3.3 Discussion of the results

The following factors cause the coordination failure:

- complementary projects
- incomplete information about the other donor's budget
- simultaneous decisions
- discrete investments

The first three factors are needed for the coordination failure to occur and the fourth factor exacerbates it.

The donors' projects are complementary. The donor's utility depends also on the number of projects financed by the other donor. If each donor cared only about their own projects and not about the general state of the sector, there would be no need for coordination and no coordination failure.

If the donors have complete information about the other donor's budget, the Nash equilibrium allocation (Table 1) is efficient (from the donors' point of view). In reality the donors are not fully aware of the other donors' activities and budgets and therefore complete information is not the relevant case to be analyzed.

If the donors move sequentially rather than simultaneously, there is no coordination failure. If donor 1 takes leadership and goes first and donor 2 invests only after observing donor 1's decisions, the allocation would be efficient. However, if the donors have different preferences sequential decisions are not a complete answer. Also in practise there is often incomplete information about the other donors' activities even ex post and therefore sequentially would not solve the information problem.

If the funds were fully divisible and the donor had Cobb-Douglas utility functions with no sufficiency level for either sector, allocation would be efficient even with incomplete information. With Cobb-Douglas utility the optimal allocation is to spend a certain proportion of the budget in each sector. Then if each donor allocates funds exactly in proportion to their priorities, the aggregate allocation would be efficient. If there is a minimum amount of funding needed in either sector (which is quite realistic in a developing country) or the donors utility functions have some other functional form for which the optimal allocation is not proportional to the budget, coordination failure occurs even if funds are fully divisible. In any case the lumpiness of funds exacerbates the coordination failure. Margins are large. In our model coordination failure is not just marginal but allocations like (4,0) and (0,2) occur in equilibrium and with a larger number of donors the misallocation can be much worse. In reality most of foreign aid is channeled as project aid which is discrete.

How would learning in a dynamic game help the coordination failure? The donors know each other's preferences and can calculate equilibrium sharing strategies and there is no need for learning the other donor's strategy. Budget size varies in every round of the game and therefore there is limited scope for learning (although the donors will learn who are the big players and who are the small players). What they can learn is the extent of misallocation after each round. Say the equilibrium of the first round is such that sector 2 does not get any funds although the aggregate budget is large. Ex post the donors learn the aggregate allocation and agree that there is a gap in services: sector 2 should get more resources. In the next round the donors have revised priorities and wish to allocate more funds towards sector 2. The only difference to the first round of the game is that the priorities are different. Coordination failure (of a different type) can occur again. Dynamic game does not solve the coordination failure.

4 Active recipient

In this section we analyze allocation game between the recipient country, R, and one donor, D. We modify the model to allow for heterogeneous agents. R's utility function is denoted by $u_R(\alpha, \beta)$ and D's by $u_D(\alpha, \beta)$. Similarly the probability distribution for R's budget is denoted by p_n^R and D's by p_n^D where $\sum_{n=0}^{3} p_n^i = 1$ for $i = R, D$.

D has the same preferences as in Section 3, that is D prefers sector 1. R's preferences are described by θ. $\theta = 0$ denotes that R has similar preferences to D. $\theta = \frac{1}{2}$ means that R values the sectors equally. $\theta = 1$ denotes that R prefers sector 2 and has the opposite preferences to D, i.e. R prefers sector 2 to sector 1 as strongly as D prefers sector 1 to sector 2. As Section 3 analyzed allocation games between parties that have similar preferences we now concentrate on $\theta = \frac{1}{2}$ (*different preferences*) and $\theta = 1$ (*opposite preferences*). Recipient government may not agree with donors about the actions that will promote welfare among its population or they may not prioritize 'pro-poor' spending because the poor have weak political voice.

The properties of R's utility functions are formalized in Assumptions 3 and 4.

Assumption 3. For $\theta = \frac{1}{2}$ (i) $\Delta u_R(\alpha, \beta) < 0$ if and only if $\alpha \geq \beta$.

(ii) $\Delta u_R(\alpha, \beta) = 0$ if and only if $\alpha = \beta - 1$.

(iii) $\Delta u_R(\alpha, \beta) > 0$ if and only if $\alpha < \beta - 1$.

Assumption 4. For $\theta = 1$ (i) $\Delta u_R(\alpha, \beta) < 0$ if and only if $\beta - \alpha < 3$.

(ii) $\Delta u_R(\alpha, \beta) > 0$ if and only if $\beta - \alpha \geq 3$.

According to Assumption 3 when R values the sectors equally reallocating a project to sector 1 increases R's utility if and only if it makes the allocation more equal. Assumption 4 says that when the agents have the opposite preferences reallocating a project to R's lower priority sector 1 reduces R's utility (property (i)) unless the original allocation was too much in favour of sector 2 (property (ii)).[11]

Allocation decisions are sequential. D allocates funds to the sectors first and R moves second. We analyze firstly complete information where R observes D's choice before his decision. Secondly, we analyze incomplete information where R moves without knowing D's action or budget. This can be modelled as a simultaneous move game with incomplete information.

4.1 Different preferences

4.1.1 Complete information

We start the analysis from $\theta = \frac{1}{2}$ and complete information. D prefers sector 1 while R values the sectors equally.

Suppose D invests α in sector 1 and the remaining $(n_D - \alpha)$ in sector 2. R observes D's choice and then aims to balance the resources in the sectors. R does not have the funds to equalize the resources if and only if $|(n_D - \alpha) - \alpha| > n_R$. Then R invests all of n_R in the sector with less funds. D would naturally choose to fund sector 1 more generously and then R invests all his funds in sector 2. The resulting allocation is $(\alpha, n_R + n_D - \alpha)$. Given

[11] Notice that although R prefers sector 2 to sector 1 as strongly as D prefers sector 1 to sector 2 Assumption 2 and 4 look somewhat different. The reason is that we have defined $\Delta u_i(\alpha, \beta)$ as reallocation of one project towards D's priority but R's lower priority sector 1.

this D chooses α to maximize his utility and thus chooses $\alpha = n_D$ (unless the budgets are (3,0)). The subgame perfect Nash equilibrium allocation is (n_D, n_R) if and only if $n_D > n_R$ and $(n_D - n_R) < 3$. For $n_D = 3$ and $n_R = 0$ the Nash equilibrium allocation is (2,1).

R has the funds to balance the resources if and only if $n_D \leq n_R$. If D has funded α projects in sector 1, R will fund β projects in sector 1 so that

$$\alpha + \beta = n_D + n_R - \alpha - \beta \Leftrightarrow \beta = \frac{n_D + n_R - 2\alpha}{2}$$

for even $(n_D + n_R)$. The resulting allocation is $\left(\frac{n_D+n_R}{2}, \frac{n_D+n_R}{2}\right)$. For odd $(n_D + n_R)$ R is indifferent between choosing $\beta = \frac{n_D+n_R-2\alpha-1}{2}$ and $\beta = \frac{n_D+n_R-2\alpha+1}{2}$. The resulting allocation is either $\left(\frac{n_D+n_R+1}{2}, \frac{n_D+n_R-1}{2}\right)$ or $\left(\frac{n_D+n_R-1}{2}, \frac{n_D+n_R+1}{2}\right)$. When $n_D \leq n_R$ the subgame perfect Nash equilibrium allocation is the same whatever α D chooses. Therefore there are multiple equilibria though the equilibrium aggregate allocation is unique (or in the case of odd aggregate budget there are two equilibrium allocations).

The subgame perfect Nash equilibrium allocations are given in Table 8. D's budget is given vertically and R's budget horizontally.

Table 8. Complete Information

	0	1	2	3
0	(0,0)	(1,0),(0,1) D	(1,1) D	(2,1),(1,2) D
1	(1,0)	(1,1) D	(2,1), (1,2) D	(2,2) D
2	(2,0) R	(2,1)	(2,2) D	(3,2), (2,3) D
3	(2,1)	(3,1) R	(3,2)	(3,3) D

When R has a larger budget than D, D cannot affect the final allocation. Whatever D does, R will balance the resources between the sectors. Without aid R would share his budget equally between the two sectors. D wishes to raise funding in his priority sector 1 but cannot do that because R responds by shifting funds away from sector 1. This is called the aid fungibility problem. When a donor builds e.g. a hospital, the recipient – who would have built that hospital anyhow – shifts his funding away from the health sector and builds a road instead or at worst increases military expenses. Therefore, the donor is not financing a hospital at the margin. In this case D could as well give the aid in the form of budget support to R and allow R to make all the allocation decisions.

When D has a larger budget than R, the agents specialize. D invests all his funds in his priority sector 1. Although R values the sectors equally, he only invests in sector 2. This is because D has such a strong preference for sector 1. Without aid R would have funded the sectors equally but given the aid R only invests in sector 2. Again aid is fungible. In reality often when the donors start funding the health and the education sectors, the recipient government moves away from these sectors and shifts funds to other sectors.

D in Table 8 denotes that there is misallocation from D's point of view (respectively for R). The resulting allocation is unfavorable to D when R has a relatively large budget and can match D's investment, in total 9 eventualities. Only in two eventualities is there R-misallocation when D has a large budget relative to R.

4.1.2 Incomplete information

With incomplete information R does not observe D's allocation or budget when making his choice. Therefore we have a simultaneous move game with incomplete information. Incomplete information introduces coordination failure between R and D. Not only is aid fungible but equilibrium allocation can be such that both D and R regret it ex post and would wish to change the allocation in the same way.

Complete specialization is the first type of Bayesian equilibrium we have. D allocates all the funds to his priority sector 1 and R invests only in sector 2. Sharing strategy $s_i = (\alpha, \beta, \gamma)$ is defined as in the previous section and denotes how many projects i allocates in sector 2.

Proposition 7 *Complete specialization* $s_D^* = (0,0,0)$ *and* $s_R^* = (1,2,3)$ *is a Bayesian equilibrium if*

(i) $p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{+}{\Delta u_D}(2,3) + p_3^R \underset{+}{\Delta u_D}(2,4) > 0,$

(ii) $p_0^D \underset{+}{\Delta u_R}(0,2) + p_1^D \underset{0}{\Delta u_R}(1,2) + p_2^D \underset{-}{\Delta u_R}(2,2) + p_3^D \underset{-}{\Delta u_R}(3,2) < 0$ *and*

(iii) $p_0^D \underset{+}{\Delta u_R}(0,3) + p_1^D \underset{+}{\Delta u_R}(1,3) + p_2^D \underset{0}{\Delta u_R}(2,3) + p_3^D \underset{-}{\Delta u_R}(3,3) < 0.$

Proof. See the Appendix.

It is obvious that D has very strong incentives to specialize in his priority sector 1 given R only funds sector 2. The only eventuality when D would invest in sector 2 is when he has a budget of 3 and R has a zero budget. If the probability of this eventuality, p_0^R, is small enough, D specializes completely in sector 1 as condition (i) in Proposition 7 states.

R aims for equal resources in both sectors. D investing all the funds in sector 1 gives good incentives for R to specialize in sector 2. Only if R has a large budget relative to D, would R invest in sector 1. If D is likely to have a large budget (2 or 3), he is able to deal with sector 1 on his own and R can concentrate on sector 2 as conditions (*ii*) and (*iii*) in Proposition 7 require.

Corollary 6 gives sufficient conditions for the conditions in Proposition 7 to be satisfied.

Corollary 6 *Complete specialization* $s_D^* = (0,0,0)$ *and* $s_R^* = (1,2,3)$ *is a Bayesian equilibrium if* $p_0^R \to 0$ *and* $p_0^D = p_1^D \to 0$.

Complete specialization occurs when R is unlikely to have a zero budget and D is likely to have a large budget (2 or 3). We have explained above the intuition for this result.

The resulting allocation is given in Table 9. D's budget is given vertically and R's budget horizontally.

Table 9. Complete Specialization

	0	1	2	3
0	(0,0)	(0,1) D	(0,2)**	(0,3)**
1	(1,0)	(1,1) D	(1,2) D	(1,3) **
2	(2,0) R	(2,1)	(2,2) D	(2,3) D
3	(3,0)*	(3,1) R	(3,2)	(3,3) D

Compared to complete information we have coordination failure in four eventualities. Coordination failure means that there is misallocation from both D's and R's point of view and they would like to change the allocation to the same direction. Sector 1 is overfunded in one eventuality marked by * and underfunded in three eventualities marked by **. Coordination failure occurs when one party has a large budget compared to the other party, in particular when R's budget is large compared to D's budget.

With complete specialization most of the coordination failure occurs because R invests too much in sector 2. If R specializes less in sector 2 this coordination failure could be eliminated. The next proposition shows when this is an equilibrium.

Proposition 8 $s_D^* = (0,0,0)$ *and* $s_R^* = (1,2,2)$ *is a Bayesian equilibrium if*

$(i)\ p_0^R \underset{-}{\Delta u_D(2,1)} + p_1^R \underset{+}{\Delta u_D(2,2)} + p_2^R \underset{+}{\Delta u_D(2,3)} + p_3^R \underset{+}{\Delta u_D(3,3)} > 0,$

$(ii)\ p_0^D \underset{+}{\Delta u_R(0,2)} + p_1^D \underset{0}{\Delta u_R(1,2)} + p_2^D \underset{-}{\Delta u_R(2,2)} + p_3^D \underset{-}{\Delta u_R(3,2)} < 0$ *and*

$(iii)\ p_0^D \underset{+}{\Delta u_R(0,3)} + p_1^D \underset{+}{\Delta u_R(1,3)} + p_2^D \underset{0}{\Delta u_R(2,3)} + p_3^D \underset{-}{\Delta u_R(3,3)} > 0.$

Proof. See the Appendix.

Now D specializes completely in sector 1 as in Proposition 7 while R shifts one project to sector 1 when he has a budget of 3. None of the conditions in Proposition 8 is particularly stringent. The main change from Proposition 7 is that condition (iii) changes the sign.[12] R prefers to invest one project in sector 1 when his budget is 3 if D is unlikely to have a budget of 3. If D had a budget of 3, he would invest it all in sector 1 and an additional project by R would make sector 1 overfunded from R's point of view.

Corollary 7 gives sufficient conditions for all the conditions in Proposition 8 to be satisfied.

Corollary 7 $s_D^* = (0,0,0)$ *and* $s_R^* = (1,2,2)$ *is a Bayesian equilibrium if* $p_0^R \to 0$ *and* $p_0^D = p_3^D \to 0$.

D has now an intermediary budget (1 or 2) and smaller than in Corollary 6. Therefore R, aiming at balance, shifts some funds to sector 1 as D is less able to deal with it on his own. R's changed strategy changes the outcome in the following way:

Table 10

	0	1	2	3
0	(0,0)	(0,1) D	(0,2)**	(1,2)
1	(1,0)	(1,1) D	(1,2) D	(2,2) D
2	(2,0) R	(2,1)	(2,2) D	(3,2)
3	(3,0)*	(3,1) R	(3,2)	(4,2) R

Compared to complete specialization two eventualities of coordination failure are eliminated.

One more eventuality of coordination failure could be eliminated if R invested in sector 1 also from $n_R = 2$. Proposition 9 finds the sufficient conditions for this equilibrium.

[12] Condition (ii) is the same as in Proposition 7 and (i) differs only by the last term.

Proposition 9 *$s_D^* = (0,0,0)$ and $s_R^* = (1,1,2)$ is a Bayesian equilibrium if*

(i) $p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{-}{\Delta u_D}(3,2) + p_3^R \underset{+}{\Delta u_D}(3,3) > 0,$

(ii) $p_0^D \underset{+}{\Delta u_R}(0,2) + p_1^D \underset{0}{\Delta u_R}(1,2) + p_2^D \underset{-}{\Delta u_R}(2,2) + p_3^D \underset{-}{\Delta u_R}(3,2) > 0$ *and*

(iii) $p_0^D \underset{+}{\Delta u_R}(0,3) + p_1^D \underset{+}{\Delta u_R}(1,3) + p_2^D \underset{0}{\Delta u_R}(2,3) + p_3^D \underset{-}{\Delta u_R}(3,3) > 0.$

Proof. See the Appendix.

The main change compared to Proposition 8 is that condition (ii) changes the sign.[13] Condition (ii) is for R to prefer sharing budget 2 between the sectors. R prefers sharing unless D has a large budget (2 or 3). D invests all the funds in sector 1 and if this investment is large R prefers not to share the resources between the sectors as it would leave sector 2 underfunded from R's point of view.

Corollary 8 gives sufficient conditions for $(i) - (iii)$ in Proposition 9 to hold.

Corollary 8 *$s_D^* = (0,0,0)$ and $s_R^* = (1,1,2)$ is a Bayesian equilibrium if $p_0^R = p_2^R \to 0$ and $p_2^D = p_3^D \to 0$.*

Compared to Corollary 7 D has an even smaller budget (0 or 1) and therefore R shifts even more funds towards sector 1 to equalize resources.

Corollaries 6 - 8 show that when D specializes in his priority sector 1, R concentrates on funding sector 2 but gives the more resources to sector 1 the smaller is D's expected budget.[14]

What is different to Corollaries 6 and 7 is that in addition to p_0^R also p_2^R is required to be negligible. To understand this we examine condition (i) in Proposition 9 which is for D to prefer investing 3 projects in sector 1. We already know why it does not hold for p_0^R large. Why it does not hold for p_2^R large is the following. If it is very likely that R has a budget of 2 which he

[13] Condition (iii) is the same as in Proposition 8 and (i) differs by the third term.

[14] In fact, we could continue this line of analysis for the smallest possible budget for D, i.e. $p_0^D \to 1$. Then R would follow his "monopoly" strategy. R shares the resources evenly between the sectors if his budget is even. If R's budget is odd, he is indifferent between which sector gets the additional project. R is then indifferent between strategies $s_R = (0,1,1)$, $s_R = (1,1,1)$, $s_R = (0,1,2)$ and $s_R = (1,1,2)$. We could then check when these strategies form a Bayesian equilbrium with $s_D = (0,0,0)$. (Proposition 9 already deals with $s_R = (1,1,2)$.) This is unnecessarily complicated and does not add much to our results. We therefore do not proceed with this.

would share between the sectors and D funds 3 projects in sector 1, sector 1 would be overfunded even from D's point of view. Therefore $s_D = (0,0,0)$ is not a best reply to $s_R = (1,1,2)$ if p_2^R is large.

The allocation resulting from the equilibrium of Proposition 9 is given in Table 11. Compared to Table 10 one event of coordination failure is eliminated but one event of overfunding of sector 1 is introduced (the event explained in the previous paragraph with budgets (3,2)).

Table 11

	0	1	2	3
0	(0,0)	(0,1) D	(1,1) D	(1,2) D
1	(1,0)	(1,1) D	(2,1)	(2,2) D
2	(2,0) R	(2,1)	(3,1) R	(3,2)
3	(3,0)*	(3,1) R	(4,1) *	(4,2) R

Now we move on to examining equilibria where D does not fully specialize in his priority sector 1 but invests one project in sector 2 from a large budget, i.e. D follows a sharing strategy $s_D = (0,0,1)$. We can prove that we can have only two types of Bayesian equilibria with $s_D = (0,0,1)$: one where $s_R = (1,2,2)$ and another where $s_R = (1,1,2)$.

Proposition 10 $s_D^* = (0,0,1)$ *and* $s_R^* = (1,2,2)$ *is a Bayesian equilibrium if*

(i) $p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{+}{\Delta u_D}(2,3) + p_3^R \underset{+}{\Delta u_D}(3,3) < 0$ *and*

(ii) $p_0^D \underset{+}{\Delta u_R}(0,2) + p_1^D \underset{0}{\Delta u_R}(1,2) + p_2^D \underset{-}{\Delta u_R}(2,2) + p_3^D \underset{0}{\Delta u_R}(2,3) < 0.$

Proof. See the Appendix.

In this equilibrium the agents specialize in different sectors and invest one project in the other sector from a large budget of 3. Condition (i) is stringent; only one of the four terms has the same sign as the whole expression. It is for D to prefer investing one project in sector 2 when his budget is 3. D will do that only if it is very likely that otherwise sector 2 does not get any resources, which is the case when R has a zero budget. Corollary 9 gives the sufficient conditions for both of the conditions in Proposition 10 to be fulfilled.

Corollary 9 $s_D^* = (0,0,1)$ *and* $s_R^* = (1,2,2)$ *is a Bayesian equilibrium if* $p_0^R \to 1$ *and* $p_0^D \to 0$.

When it is very likely that R has a zero budget, D is willing to share when his budget is 3. Condition (ii) in Proposition 10 is for R to invest all of budget 2 in sector 2. As D is concentrating on sector 1, R aiming for balanced resources can invest all of budget 2 in sector 2 unless D has no resources, i.e. if p_0^D is negligible. Table 12 reports the resulting allocations.

Table 12

	0	1	2	3
0	(0,0)	(0,1) D	(0,2)**	(1,2) D
1	(1,0)	(1,1) D	(1,2) D	(2,2) D
2	(2,0) R	(2,1)	(2,2) D	(3,2)
3	(2,1)	(2,2) D	(2,3) D	(3,3) D

There is only one eventuality of coordination failure where sector 2 is overfunded.

The second possible equilibrium where D shares marginally is proved in Proposition 11.

Proposition 11 $s_D^* = (0,0,1)$ *and* $s_R^* = (1,1,2)$ *is a Bayesian equilibrium if*

(i) $p_0^R \underset{-}{\Delta u_D(2,1)} + p_1^R \underset{+}{\Delta u_D(2,2)} + p_2^R \underset{-}{\Delta u_D(3,2)} + p_3^R \underset{+}{\Delta u_D(3,3)} < 0$ *and*

(ii) $p_0^D \underset{+}{\Delta u_R(0,2)} + p_1^D \underset{0}{\Delta u_R(1,2)} + p_2^D \underset{-}{\Delta u_R(2,2)} + p_3^D \underset{0}{\Delta u_R(2,3)} > 0.$

Proof. See the Appendix.

Compared to Proposition 10 R is investing less in sector 2. Condition (i) is nearly the same as in Proposition 10: only the third term is different and as it is now negative the condition is easier to satisfy. D is more willing to invest one project in sector 2 (when his budget is 3) when R invests less in sector 2. The terms in condition (ii) are the same as in Proposition 10, only now the expression is required to be positive so that R rather shares a budget of 2 between the sectors than invests it all in sector 2. The condition is satisfied unless p_2^D is very large. Then D would invest 2 projects in sector 1 and R would prefer to match the investment in sector 2.

Corollary 10 again gives the sufficient conditions for (i) and (ii) in Proposition 11 to be fulfilled.

Corollary 10 $s_D^* = (0,0,1)$ *and* $s_R^* = (1,1,2)$ *is a Bayesian equilibrium if* $p_1^R = p_3^R \to 0$ *and* $p_2^D \to 0$.

Corollary 10 shows that D is willing to share when R's budget is either 0 or 2. It is clear why R's zero budget gives D incentives to share. Why $n_R = 2$ induces D to share is the following. R would share his budget of 2 between the sectors and an additional three projects in sector 1 would lead to too unbalanced allocation even from D's point of view. Table 13 gives the resulting allocation.

Table 13

	0	1	2	3
0	(0,0)	(0,1) D	(1,1) D	(1,2) D
1	(1,0)	(1,1) D	(2,1)	(2,2) D
2	(2,0) R	(2,1)	(3,1) R	(3,2) D
3	(2,1)	(2,2) D	(3,2)	(3,3) D

In this equilibrium coordination failure is eliminated completely. It does not, however, implement the complete information allocation as one outcome is turned unfavorable to D (budgets (3,1)) and one outcome is turned unfavorable to R (budgets (2,2)).

We have checked that the equilibria examined above in Propositions 7 - 11 (see also footnote 11) are the only pure strategy Bayesian equilibria that can exist in this allocation game. We have restricted analysis to weakly increasing sharing strategies. We observe from these results that only $s_D = (0,0,0)$ or $s_D = (0,0,1)$ can arise in a pure strategy Bayesian equilibrium. In other words, D maximally invests one project in his lower priority sector.

4.1.3 Summary

When D prefers sector 1 and R values the sectors equally D concentrates on his priority sector. D specializes fully in sector 1, i.e. follows $s_D = (0,0,0)$, when R is able to deal with sector 2 on his own (p_0^R is negligible). While R shifts the more funds to sector 1 the lower are D's expected resources. D shares marginally, i.e. follows $s_D = (0,0,1)$, when R may not be able to deal with sector 2 on his own ($p_0^R > 0$). Then only two equilibria are possible and there is not as clear pattern but R again shifts funds between the sectors so that expected allocation would be equalized.

Aid fungibility problem exists also with incomplete information: R responds to D's strategy. However, incomplete information gives some power to D. With complete information and $n_D \leq n_R$ D was indifferent between any sharing strategy. His choice had no effect on the final allocation because R could match his investments since he has a larger budget. Now D's choice matters because of incomplete information R cannot fully match D's investment. D can therefore influence the final allocation. On the other hand, incomplete information introduces coordination failure and the final allocation can be unfavorable to both D and R.

4.2 Opposite preferences

4.2.1 Complete information

Finally we analyze $\theta = 1$. D prefers sector 1 and R prefers sector 2. With complete information D invests all the funds in sector 1 unless budgets are (3,0) in which case D invests one project in sector 2. Similarly R invests all the funds in sector 2 unless the budgets are (0,3). The resulting allocations are:

Table 14. Complete Information, Opposite Preferences

	0	1	2	3
0	(0,0)	(0,1) D	(0,2) D	(1,2) D
1	(1,0) R	(1,1) D,R	(1,2) D	(1,3) D
2	(2,0) R	(2,1) R	(2,2) D,R	(2,3) D
3	(2,1) R	(3,1) R	(3,2) R	(3,3) D,R

When the parties have completely opposite preferences, there is always misallocation from someone's point of view. The party with a relatively large budget is able to enforce an allocation that is favorable to him. Timing of the moves makes no difference. Simultaneous and sequential decisions lead to the same allocation with complete information. When the resources are equally divided between the sectors both D and R view it as a misallocation. However, it is not coordination failure as D prefers to reallocate towards sector 1 and R prefers to reallocate towards sector 2.

4.2.2 Incomplete information

From the complete information case we know that D and R will not share from budget sizes 1 and 2 and wish to invest at most one project in the lower priority sector when their budget size is 3. Therefore the only equilibria that can exist are:

(i) Symmetric sharing equilibrium: both D and R invest one project in the lower priority sector from budget size 3.

(ii) Symmetric no sharing equilibrium (complete specialization): both D and R invest all the funds in their most preferred sector.

(iii) Asymmetric equilibria: D shares but R does not, R shares but D does not.

With similar and different preferences a variety of equilibria can exist but with the opposite preferences only these three equilibria are possible. Therefore we can derive both the necessary and the sufficient conditions, which we do in Propositions 13 - 15.

Proposition 12 $s_D^* = (0,0,0)$ *and* $s_R^* = (1,2,3)$ *is a Bayesian equilibrium if and only if* $p_0^R < \widetilde{p}_0^R$ *and* $p_0^D < \widetilde{p}_0^D$ *where* $\widetilde{p}_0^R = -\frac{p_1^R \Delta u_D(2,2) + p_2^R \Delta u_D(2,3) + p_3^R \Delta u_D(2,4)}{\Delta u_D(2,1)}$ *and* $\widetilde{p}_0^D = -\frac{p_1^D \Delta u_R(1,3) + p_2^D \Delta u_R(2,3) + p_3^D \Delta u_R(3,3)}{\Delta u_R(0,3)}$.

Proposition 13 (i) $s_D^* = (0,0,1)$ *and* $s_R^* = (1,2,3)$ *is a Bayesian equilibrium if and only if* $p_0^R > \widetilde{p}_0^R$ *and* $p_0^D < \widehat{p}_0^D$ *where* $\widehat{p}_0^D = -\frac{p_1^D \Delta u_R(1,3) + p_2^D \Delta u_R(2,3) + p_3^D \Delta u_R(2,4)}{\Delta u_R(0,3)}$.

(ii) $s_D^* = (0,0,0)$ *and* $s_R^* = (1,2,2)$ *is a Bayesian equilibrium if and only if* $p_0^R < \widehat{p}_0^R$ *and* $p_0^D > \widetilde{p}_0^D$ *where* $\widehat{p}_0^R = -\frac{p_1^R \Delta u_D(2,2) + p_2^R \Delta u_D(2,3) + p_3^R \Delta u_D(3,3)}{\Delta u_D(2,1)}$.

Proposition 14 $s_D^* = (0,0,1)$ *and* $s_R^* = (1,2,2)$ *is a Bayesian equilibrium if and only if* $p_0^R > \widehat{p}_0^R$ *and* $p_0^D > \widehat{p}_0^D$.

Proofs. See the Appendix.

Propositions 12 - 14 help us to construct Figure 1. It is intuitive to assume that $\widetilde{p}_0^i > \widehat{p}_0^i$. For D this is implied by $\Delta \underset{-}{u_R}(3,3) < \Delta \underset{-}{u_R}(2,4)$. In other words, shifting resources away from R's priority sector 2 is reducing R's welfare less when sector 2 is originally better funded than sector 1 than when the sectors are equally funded. $\widetilde{p}_0^R > \widehat{p}_0^R$ requires $\Delta \underset{+}{u_D}(2,4) > \underset{+}{u_D}(3,3)$. When we shift resources towards D's priority sector 1 D's utility increases more when sector 1 is originally funded less than sector 2 than when we start from equal funding.

In Figure 1 we observe that for very high $p_0^j > \widetilde{p}_0^j$ agent i shares whatever is j's strategy. If it is very likely that agent j is not able to fund his priority sector, then agent i will share to avoid too unbalanced allocation.

For somewhat lower p_0^j, $\widehat{p}_0^j < p_0^j < \widetilde{p}_0^j$, agent i shares only if agent j shares too. When agent j shares, the expected funds in i's priority sector are increased. To achieve a more balanced allocation agent i in return shifts funds towards j's priority sector since it is fairly likely that j has a zero budget. This explains the multiplicity of equilibria: either both agents share or neither does in three regions in Figure 1.

Finally, if both p_0^D and p_0^R are very low neither agent shares as both agents can fund their priority sectors.

We have therefore identified two forces that give incentives for sharing when the agents have the opposite preferences:

(i) If it is very likely that agent j is unable to fund his priority sector, i.e. p_0^j is high, then agent i will share.

(ii) If agent j shares, agent i's incentives to share are increased as expected funds are shifted from j's priority sector towards i's priority sector.

Tables 15–18 report the equilibrium allocations with opposite preferences. Some of the allocations are the same as in the last subsection. However, complete information case differs and it is the comparison to that that determines what is coordination failure. Note that the definition of coordination failure is that there is an allocation that Pareto dominates the equilibrium allocation. There are allocations that both D and R view as misallocations but it is not coordination failure as each would like to change it in favor of their preferred sector.

Table 15. Complete Specialization

	0	1	2	3
0	(0,0)	(0,1) D	(0,2) D	(0,3)*
1	(1,0) R	(1,1) D,R	(1,2) D	(1,3) D
2	(2,0) R	(2,1) R	(2,2) D,R	(2,3) D
3	(3,0)*	(3,1) R	(3,2) R	(3,3) D,R

Table 16. D Sharing Equilibrium

	0	1	2	3
0	(0,0)	(0,1) D	(0,2) D	(0,3)*
1	(1,0) R	(1,1) D,R	(1,2) D	(1,3) D
2	(2,0) R	(2,1) R	(2,2) D,R	(2,3) D
3	(2,1) R	(2,2) D,R	(2,3) D	(2,4) D

Table 17. R Sharing Equilibrium

	0	1	2	3
0	(0,0)	(0,1) D	(0,2) D	(1,2) D
1	(1,0) R	(1,1) D,R	(1,2) D	(2,2) D,R
2	(2,0) R	(2,1) R	(2,2) D,R	(3,2) R
3	(3,0)*	(3,1) R	(3,2) R	(4,2) R

Table 18. Symmetric Sharing Equilibrium

	0	1	2	3
0	(0,0)	(0,1) D	(0,2) D	(1,2) D
1	(1,0) R	(1,1) D,R	(1,2) D	(2,2) D,R
2	(2,0) R	(2,1) R	(2,2) D,R	(3,2) R
3	(2,1) R	(2,2) D,R	(2,3) D	(3,3) D,R

With the opposite preferences there are at most two cases of coordination failure. In an equilibrium where both agents share coordination failure is eliminated completely.

Although coordination failure occurs in very few cases allocations do change from the complete information case. The agent who shares does that not only when the other agent's budget is 0 but for the other budget sizes too. Therefore he shares too much (from complete information perspective) and turns some allocations unfavorable to himself.

4.2.3 Summary

When the agents have the opposite preferences, they each specialize in their priority sector and invest maximally one project in the lower priority sector. Some coordination failure can occur in equilibrium but its extent is minimal.

5 Conclusions

This paper has analyzed three versions of a foreign aid allocation game with increasingly different preferences between the agents. It first analyzed an allocation game with two identical donors who prioritize sector 1, and found that incomplete information can lead to coordination failure. There are two types of coordination failure. First, the lower priority sector does not get enough funds as both donors concentrate too much on the priority sector (undersharing). Second, there may be gaps in services in the priority sector due to overfunding of the lower-priority sector (oversharing). When the other donor is expected to have a small budget, both donors concentrate on the priority sector and share marginally. This can lead to either undersharing or oversharing. With high expected budgets, the donors specialize. One donor fully or partially specializes in the priority sector while the other donor mainly invests in the lower-priority sector, resulting in oversharing in some eventualities.

The analysis then introduced an active recipient country with equal preferences for both sectors. In the allocation game, the donor concentrates on funding his priority sector. The recipient mainly funds the donor's lower-priority sector and shifts the funds between the sectors according to the donor's ability to finance his priority sector. In other words, aid is fungible. However, with incomplete information, the recipient cannot fully match the donor's investment and the donor can affect the final allocation. Therefore in some sense the aid fungibility problem is reduced by incomplete information. On the other hand, incomplete information introduces coordination failure so that the final allocation can be inferior to *both* the recipient *and* the donor.

The paper finally analyzed a case where the recipient prefers the donor's lower-priority sector, that is, the agents have opposite preferences. With these preferences each agent focuses on his priority sector and shares marginally, and there was minimal coordination failure.

Comparing the extent of coordination failure with three types of preferences (see Tables 2–18) shows that the more similar the preferences are, the more scope there is for coordination failure. With similar preferences, coordination failure occurs in 3 to 8 eventualities, with different preferences in 0 to 4 eventualities and with opposite preferences in only 0 to 2 eventualities. When similar agents pursue the common good, misallocation due to incomplete information can be considerable. When the agents have very different priorities about their activities in the recipient developing country, informational problems are minimal.

The focus of this paper has been on coordination failure among agents who run projects in a recipient developing country. The paper has not evaluated the arising equilibrium against a social welfare function but simply examined the extent of Pareto inefficient allocations among providers. The government of the recipient country may for example represent particular constituencies where the poor may not be well represented. Therefore the recipient government's utility function is not the social welfare function.

Likewise although the donors are interested in the welfare of the recipient country, they also have interests of their own (e.g., their own country or bureaucracy). Therefore we are *not* saying that social welfare is higher when agents have more preferences. We are simply saying that the inefficiency arising from incomplete information is worse when preferences are more similar.

An interesting direction to extend the analysis is to take into account that the donor's utility depends not only on the aggregate amount of funding in each sector, but also on his own contribution. Aid agencies have a need for visibility to justify their activities and therefore get more utility from their own donations. This paper's polar case is one step toward understanding the allocation of foreign aid. Furthermore, in this model even an active recipient has no power over how the donor's projects are allocated but the donor can unilaterally make decisions about his funds. A recipient with more bargaining power would be another interesting extension.

6 Appendix

6.1 Proofs for Section 3

We first lay out the basic structure of the proofs which will then be applied to prove each proposition.

Suppose donor j's equilibrium sharing strategy is $s_j^* = (\alpha, \beta, \gamma)$ where $\alpha \in \{0,1\}, \beta \in \{0,1,2\}, \gamma \in \{0,1,2\}$. From the complete information case we know that the donors never wish to have more than two projects in sector 2. s_j^* implies donor j allocates $(1-\alpha, 2-\beta, 3-\gamma)$ projects in sector 1. What is donor i's best response? We analyze each budget size for donor i separately.

If $n_i = 1$ donor i is better off investing it in sector 1 if and only if:

$$p_0 u(1,0) + p_1 u(2-\alpha, \alpha) + p_2 u(3-\beta, \beta) + p_3 u(4-\gamma, \gamma) \tag{3}$$

$$> p_0 u(0,1) + p_1 u(1-\alpha, 1+\alpha) + p_2 u(2-\beta, 1+\beta) + p_3 u(3-\gamma, 1+\gamma)$$

Equation (3) simplifies to:

$$p_0 \Delta u(0,1) + p_1 \Delta u(1-\alpha, 1+\alpha) + p_2 \Delta u(2-\beta, 1+\beta) + p_3 \Delta u(3-\gamma, 1+\gamma) > 0 \tag{4}$$

The first entry in donor i's best response is zero, i.e. $s_i = (0, \beta', \gamma')$, if and only if equation (4) holds. The first entry is one, $s_i = (1, \beta', \gamma')$, if and only if equation (4) does not hold.

Given $n_i = 2$ donor i is better off investing all the funds in sector 1 rather than sharing the funds between the sectors if and only if:

$$p_0 \Delta u(1,1) + p_1 \Delta u(2-\alpha, 1+\alpha) + p_2 \Delta u(3-\beta, 1+\beta) + p_3 \Delta u(4-\gamma, 1+\gamma) > 0 \tag{5}$$

Given $n_i = 2$ donor i prefers sharing the funds between the sectors to investing all the funds in sector 2 if and only if:

$$p_0 \Delta u(0,2) + p_1 \Delta u(1-\alpha, 2+\alpha) + p_2 \Delta u(2-\beta, 2+\beta) + p_3 \Delta u(3-\gamma, 2+\gamma) > 0 \tag{6}$$

The second entry in donor i's best response is zero, i.e. $s_i = (\alpha', 0, \gamma')$, if both (5) and (6) hold. The best response is $s_i = (\alpha', 1, \gamma')$ if and only if (5) does not hold and (6) holds. Finally, the second entry is 2, i.e. $s_i = (\alpha', 2, \gamma')$, if neither (5) nor (6) hold.

Finally, we analyze $n_i = 3$. Donor i prefers allocating 3 rather than 2 projects in sector 1 if and only if:

$$p_0 \Delta u(2,1) + p_1 \Delta u(3-\alpha, 1+\alpha) + p_2 \Delta u(4-\beta, 1+\beta) + p_3 \Delta u(5-\gamma, 1+\gamma) > 0 \tag{7}$$

Donor i prefers funding 1 rather than 2 projects in sector 2 if and only if:

$$p_0 \Delta u(1,2) + p_1 \Delta u(2-\alpha, 2+\alpha) + p_2 \Delta u(3-\beta, 2+\beta) + p_3 \Delta u(4-\gamma, 2+\gamma) > 0 \tag{8}$$

Accordingly, the third entry is zero if both (7) and (8) hold. The best response is $s_i = (\alpha', \beta', 1)$ if and only if (7) does not hold and (8) holds. Finally, the third entry is 2, i.e. $s_i = (\alpha', \beta', 2)$, if neither (7) nor (8) hold.

We use equations (4) – (8) in all the proofs by inserting the appropriate values for α, β and γ.

Proof of Proposition 2.

$s_i^* = (0, 0, 1)$ for $i = 1, 2$ is Bayesian equilibrium if equations $(4), (5), (6)$ and (8) hold and (7) does not hold for $\alpha = 0$, $\beta = 0$ and $\gamma = 1$.

$$p_0 \underset{+}{\Delta u}(0,1) + p_1 \underset{+}{\Delta u}(1,1) + p_2 \underset{-}{\Delta u}(2,1) + p_3 \underset{+}{\Delta u}(2,2) > 0 \tag{9}$$

$$p_0 \underset{+}{\Delta u}(1,1) + p_1 \underset{-}{\Delta u}(2,1) + p_2 \underset{-}{\Delta u}(3,1) + p_3 \underset{-}{\Delta u}(3,2) > 0 \tag{10}$$

$$p_0 \underset{+}{\Delta u}(0,2) + p_1 \underset{+}{\Delta u}(1,2) + p_2 \underset{+}{\Delta u}(2,2) + p_3 \underset{+}{\Delta u}(2,3) > 0 \tag{11}$$

$$p_0 \underset{-}{\Delta u}(2,1) + p_1 \underset{-}{\Delta u}(3,1) + p_2 \underset{-}{\Delta u}(4,1) + p_3 \underset{-}{\Delta u}(4,2) < 0 \tag{12}$$

$$p_0 \underset{+}{\Delta u}(1,2) + p_1 \underset{+}{\Delta u}(2,2) + p_2 \underset{-}{\Delta u}(3,2) + p_3 \underset{+}{\Delta u}(3,3) > 0 \tag{13}$$

(11) and (12) hold unambiguously. Therefore $s_i^* = (0,0,1)$ for $i = 1,2$ is Bayesian equilibrium if equations $(9), (10)$ and (13) hold.

Q.E.D.

Proof of Proposition 3.

$s_i^* = (0,1,1)$ for $i = 1,2$ is Bayesian equilibrium if and only if equations $(4), (6)$ and (8) hold and (5) and (7) do not hold for $\alpha = 0$, $\beta = 1$ and $\gamma = 1$.

$$p_0 \underset{+}{\Delta u(0,1)} + p_1 \underset{+}{\Delta u(1,1)} + p_2 \underset{+}{\Delta u(1,2)} + p_3 \underset{+}{\Delta u(2,2)} > 0 \tag{14}$$

$$p_0 \underset{+}{\Delta u(1,1)} + p_1 \underset{-}{\Delta u(2,1)} + p_2 \underset{+}{\Delta u(2,2)} + p_3 \underset{-}{\Delta u(3,2)} < 0 \tag{15}$$

$$p_0 \underset{+}{\Delta u(0,2)} + p_1 \underset{+}{\Delta u(1,2)} + p_2 \underset{+}{\Delta u(1,3)} + p_3 \underset{+}{\Delta u(2,3)} > 0 \tag{16}$$

$$p_0 \underset{-}{\Delta u(2,1)} + p_1 \underset{-}{\Delta u(3,1)} + p_2 \underset{-}{\Delta u(3,2)} + p_3 \underset{-}{\Delta u(4,2)} < 0 \tag{17}$$

$$p_0 \underset{+}{\Delta u(1,2)} + p_1 \underset{+}{\Delta u(2,2)} + p_2 \underset{+}{\Delta u(2,3)} + p_3 \underset{+}{\Delta u(3,3)} > 0 \tag{18}$$

$(14), (16), (17)$ and (18) hold unambiguously. Therefore $s_i^* = (0,1,1)$ for $i = 1,2$ is Bayesian equilibrium if and only if equation (15) holds. Q.E.D.

Proof of Proposition 4.

Donor i's best response to $s_j = (0,1,1)$ is $s_i = (0,0,1)$ if $(14), (16) - (18)$ hold (and they hold unambiguously) and

$$p_0 \underset{+}{\Delta u(1,1)} + p_1 \underset{-}{\Delta u(2,1)} + p_2 \underset{+}{\Delta u(2,2)} + p_3 \underset{-}{\Delta u(3,2)} > 0 \tag{19}$$

Donor j's best response to $s_i = (0,0,1)$ is $s_j = (0,1,1)$ if and only if $(9), (11) - (13)$ hold and

$$p_0 \underset{+}{\Delta u(1,1)} + p_1 \underset{-}{\Delta u(2,1)} + p_2 \underset{-}{\Delta u(3,1)} + p_3 \underset{-}{\Delta u(3,2)} < 0 \tag{20}$$

Equations (11) and (12) hold unambiguously.

$s_i^* = (0,0,1)$ and $s_j^* = (0,1,1)$ is a Bayesian equilibrium if equations $(9), (13), (19)$ and (20) and hold. Equations (19) and (20) can be combined as:

$$-p_2\underset{+}{\Delta u}(2,2) < p_0\underset{+}{\Delta u}(1,1) + p_1\underset{-}{\Delta u}(2,1) + p_3\underset{-}{\Delta u}(3,2) < -p_2\underset{-}{\Delta u}(3,1) \tag{21}$$

Notice that $-p_2\underset{+}{\Delta u}(2,2) < -p_2\underset{-}{\Delta u}(3,1)$ and therefore (21) is feasible. The sufficient conditions are therefore (9), (13) and (21).

Q.E.D.

Proof of Corollary 3.

Equations (9), (13), (19) and (20) and are satisfied if the probability distribution is uniform ($p_0 = p_1 = p_2 = p_3$) and the following conditions hold:

$$\underset{+}{\Delta u}(1,1) + \underset{+}{\Delta u}(2,2) > -\underset{-}{\Delta u}(2,1) - \underset{-}{\Delta u}(3,2) \tag{22}$$

$$\underset{+}{\Delta u}(0,1) + \underset{+}{\Delta u}(1,1) + \underset{+}{\Delta u}(2,2) > -\underset{-}{\Delta u}(2,1) \tag{23}$$

$$\underset{+}{\Delta u}(1,1) < -\underset{-}{\Delta u}(2,1) - \underset{-}{\Delta u}(3,1) - \underset{-}{\Delta u}(3,2) \tag{24}$$

$$\underset{+}{\Delta u}(1,2) + \underset{+}{\Delta u}(2,2) + \underset{+}{\Delta u}(3,3) > -\underset{-}{\Delta u}(3,2) \tag{25}$$

We have moved all the negative terms to the right-hand-side of the equations. (22) implies (23) since we can rewrite (23) as:

$$\underset{+}{\Delta u}(1,1) + \underset{+}{\Delta u}(2,2) > -\underset{-}{\Delta u}(2,1) - \underset{+}{\Delta u}(0,1) \tag{26}$$

and the right-hand-side of (22) is larger than the right-hand-side of (26).

Equations (22) and (24) are satisfied if

$$-\Delta u(2,2) < \Delta u(1,1) + \Delta u(2,1) + \Delta u(3,2) < -\Delta u(3,1) \tag{27}$$

Accordingly, if equations (27) and (25) hold $s_i^* = (0,0,1)$ and $s_j^* = (0,1,1)$ is a Bayesian equilibrium. Q.E.D.

Proof of Proposition 5.

Donor i's best response to $s_j = (1,2,2)$ is $s_i = (0,0,0)$ if

$$p_0\underset{+}{\Delta u}(0,1) + p_1\underset{+}{\Delta u}(0,2) + p_2\underset{+}{\Delta u}(0,3) + p_3\underset{+}{\Delta u}(1,3) > 0 \tag{28}$$

$$p_0\underset{+}{\Delta u}(1,1) + p_1\underset{+}{\Delta u}(1,2) + p_2\underset{+}{\Delta u}(1,3) + p_3\underset{+}{\Delta u}(2,3) > 0 \tag{29}$$

$$p_0\underset{+}{\Delta u}(0,2) + p_1\underset{+}{\Delta u}(0,3) + p_2\underset{+}{\Delta u}(0,4) + p_3\underset{+}{\Delta u}(1,4) > 0 \tag{30}$$

$$p_0\underset{-}{\Delta u}(2,1) + p_1\underset{+}{\Delta u}(2,2) + p_2\underset{+}{\Delta u}(2,3) + p_3\underset{+}{\Delta u}(3,3) > 0 \tag{31}$$

$$p_0\underset{+}{\Delta u}(1,2) + p_1\underset{+}{\Delta u}(1,3) + p_2\underset{+}{\Delta u}(1,4) + p_3\underset{+}{\Delta u}(2,4) > 0 \tag{32}$$

Equations $(28) - (30)$ and (32) hold unambiguously.

Donor j's best response to $s_i = (0,0,0)$ is $s_j = (1,2,2)$ if

$$p_0\underset{+}{\Delta u}(0,1) + p_1\underset{+}{\Delta u}(1,1) + p_2\underset{-}{\Delta u}(2,1) + p_3\underset{-}{\Delta u}(3,1) < 0 \tag{33}$$

$$p_0\underset{+}{\Delta u}(1,1) + p_1\underset{-}{\Delta u}(2,1) + p_2\underset{-}{\Delta u}(3,1) + p_3\underset{-}{\Delta u}(4,1) < 0 \tag{34}$$

$$p_0\underset{+}{\Delta u}(0,2) + p_1\underset{+}{\Delta u}(1,2) + p_2\underset{+}{\Delta u}(2,2) + p_3\underset{-}{\Delta u}(3,2) < 0 \tag{35}$$

$$p_0\underset{-}{\Delta u}(2,1) + p_1\underset{-}{\Delta u}(3,1) + p_2\underset{-}{\Delta u}(4,1) + p_3\underset{-}{\Delta u}(5,1) < 0 \tag{36}$$

$$p_0\underset{+}{\Delta u}(1,2) + p_1\underset{+}{\Delta u}(2,2) + p_2\underset{-}{\Delta u}(3,2) + p_3\underset{-}{\Delta u}(4,2) < 0 \tag{37}$$

Equation (36) holds unambiguously.

$s_i^* = (0,0,0)$ and $s_j^* = (1,2,2)$ is a Bayesian equilibrium if equations (31), (33) – (35) and (37) hold. Q.E.D.

Proof of Proposition 6.

Donor i's best response to $s_j = (1,1,2)$ is $s_i = (0,0,1)$ if

$$p_0\underset{+}{\Delta u}(0,1) + p_1\underset{+}{\Delta u}(0,2) + p_2\underset{+}{\Delta u}(1,2) + p_3\underset{+}{\Delta u}(1,3) > 0 \tag{38}$$

$$p_0\underset{+}{\Delta u}(1,1) + p_1\underset{+}{\Delta u}(1,2) + p_2\underset{+}{\Delta u}(2,2) + p_3\underset{+}{\Delta u}(2,3) > 0 \tag{39}$$

$$p_0\underset{+}{\Delta u}(0,2) + p_1\underset{+}{\Delta u}(0,3) + p_2\underset{+}{\Delta u}(1,3) + p_3\underset{+}{\Delta u}(1,4) > 0 \tag{40}$$

$$p_0 \underset{-}{\Delta u}(2,1) + p_1 \underset{+}{\Delta u}(2,2) + p_2 \underset{-}{\Delta u}(3,2) + p_3 \underset{+}{\Delta u}(3,3) < 0 \tag{41}$$

$$p_0 \underset{+}{\Delta u}(1,2) + p_1 \underset{+}{\Delta u}(1,3) + p_2 \underset{+}{\Delta u}(2,3) + p_3 \underset{+}{\Delta u}(2,4) > 0 \tag{42}$$

Equations $(38)-(40)$ and (42) hold unambiguously.

Donor j's best response to $s_i = (0,0,1)$ is $s_j = (1,1,2)$ if

$$p_0 \underset{+}{\Delta u}(0,1) + p_1 \underset{+}{\Delta u}(1,1) + p_2 \underset{-}{\Delta u}(2,1) + p_3 \underset{+}{\Delta u}(2,2) < 0 \tag{43}$$

$$p_0 \underset{+}{\Delta u}(1,1) + p_1 \underset{-}{\Delta u}(2,1) + p_2 \underset{-}{\Delta u}(3,1) + p_3 \underset{-}{\Delta u}(3,2) < 0 \tag{44}$$

$$p_0 \underset{+}{\Delta u}(0,2) + p_1 \underset{+}{\Delta u}(1,2) + p_2 \underset{+}{\Delta u}(2,2) + p_3 \underset{+}{\Delta u}(2,3) > 0 \tag{45}$$

$$p_0 \underset{-}{\Delta u}(2,1) + p_1 \underset{-}{\Delta u}(3,1) + p_2 \underset{-}{\Delta u}(4,1) + p_3 \underset{-}{\Delta u}(4,2) < 0 \tag{46}$$

$$p_0 \underset{+}{\Delta u}(1,2) + p_1 \underset{+}{\Delta u}(2,2) + p_2 \underset{-}{\Delta u}(3,2) + p_3 \underset{+}{\Delta u}(3,3) < 0 \tag{47}$$

Equations (45)and (46) hold unambiguously. Equation (47) implies (41) as they differ only in the first term and the first term is negative in (41) and positive in (47).

$s_i^* = (0,0,1)$ and $s_j^* = (1,1,2)$ is a Bayesian equilibrium if equations (43), (44) and (47) are satisfied. Q.E.D.

6.2 Proofs for Section 4

6.2.1 Proofs for different preferences

The structure of the proofs is very similar to Section 3. We only need to modify to have different utility functions and probability distributions for D and R. The equivalent of equations $(4)-(8)$ for D are:

$$p_0^R \Delta u_D(0,1) + p_1^R \Delta u_D(1-\alpha, 1+\alpha) + p_2^R \Delta u_D(2-\beta, 1+\beta) + p_3^R \Delta u_D(3-\gamma, 1+\gamma) > 0 \tag{48}$$

$$p_0^R \Delta u_D(1,1) + p_1^R \Delta u_D(2-\alpha, 1+\alpha) + p_2^R \Delta u_D(3-\beta, 1+\beta) + p_3^R \Delta u_D(4-\gamma, 1+\gamma) > 0 \tag{49}$$

$$p_0^R \Delta u_D(0,2) + p_1^R \Delta u_D(1-\alpha, 2+\alpha) + p_2^R \Delta u_D(2-\beta, 2+\beta) + p_3^R \Delta u_D(3-\gamma, 2+\gamma) > 0 \tag{50}$$

$$p_0^R \Delta u_D(2,1) + p_1^R \Delta u_D(3-\alpha, 1+\alpha) + p_2^R \Delta u_D(4-\beta, 1+\beta) + p_3^R \Delta u_D(5-\gamma, 1+\gamma) > 0 \tag{51}$$

$$p_0^R \Delta u_D(1,2) + p_1^R \Delta u_D(2-\alpha, 2+\alpha) + p_2^R \Delta u_D(3-\beta, 2+\beta) + p_3^R \Delta u_D(4-\gamma, 2+\gamma) > 0 \tag{52}$$

For R we also have to take into account that he may invest 3 projects in sector 2 (equation (58)) and we have:

$$p_0^D \Delta u_R(0,1) + p_1^D \Delta u_R(1-\alpha, 1+\alpha) + p_2^D \Delta u_R(2-\beta, 1+\beta) + p_3^D \Delta u_R(3-\gamma, 1+\gamma) > 0 \tag{53}$$

$$p_0^D \Delta u_R(1,1) + p_1^D \Delta u_R(2-\alpha, 1+\alpha) + p_2^D \Delta u_R(3-\beta, 1+\beta) + p_3^D \Delta u_R(4-\gamma, 1+\gamma) > 0 \tag{54}$$

$$p_0^D \Delta u_R(0,2) + p_1^D \Delta u_R(1-\alpha, 2+\alpha) + p_2^D \Delta u_R(2-\beta, 2+\beta) + p_3^D \Delta u_R(3-\gamma, 2+\gamma) > 0 \tag{55}$$

$$p_0^D \Delta u_R(2,1) + p_1^D \Delta u_R(3-\alpha, 1+\alpha) + p_2^D \Delta u_R(4-\beta, 1+\beta) + p_3^D \Delta u_R(5-\gamma, 1+\gamma) > 0 \tag{56}$$

$$p_0^D \Delta u_R(1,2) + p_1^D \Delta u_R(2-\alpha, 2+\alpha) + p_2^D \Delta u_R(3-\beta, 2+\beta) + p_3^D \Delta u_R(4-\gamma, 2+\gamma) > 0 \tag{57}$$

$$p_0^D \Delta u_R(0,3) + p_1^D \Delta u_R(1-\alpha, 3+\alpha) + p_2^D \Delta u_R(2-\beta, 3+\beta) + p_3^D \Delta u_R(3-\gamma, 3+\gamma) > 0 \tag{58}$$

We use these equations in all the proofs by substituting in the other agent's sharing strategy and changing the sign of the equations appropriately.

Proof of Proposition 7.

D's best response to $s_R = (1,2,3)$ is $s_D = (0,0,0)$ if

$$p_0^R \underset{+}{\Delta u_D}(0,1) + p_1^R \underset{+}{\Delta u_D}(0,2) + p_2^R \underset{+}{\Delta u_D}(0,3) + p_3^R \underset{+}{\Delta u_D}(0,4) > 0 \tag{59}$$

$$p_0^R \underset{+}{\Delta u_D}(1,1) + p_1^R \underset{+}{\Delta u_D}(1,2) + p_2^R \underset{+}{\Delta u_D}(1,3) + p_3^R \underset{+}{\Delta u_D}(1,4) > 0 \tag{60}$$

$$p_0^R \underset{+}{\Delta u_D}(0,2) + p_1^R \underset{+}{\Delta u_D}(0,3) + p_2^R \underset{+}{\Delta u_D}(0,4) + p_3^R \underset{+}{\Delta u_D}(0,5) > 0 \tag{61}$$

$$p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{+}{\Delta u_D}(2,3) + p_3^R \underset{+}{\Delta u_D}(2,4) > 0 \tag{62}$$

$$p_0^R \underset{+}{\Delta u_D}(1,2) + p_1^R \underset{+}{\Delta u_D}(1,3) + p_2^R \underset{+}{\Delta u_D}(1,4) + p_3^R \underset{+}{\Delta u_D}(1,5) > 0 \tag{63}$$

Equations $(59) - (61)$ and (63) hold unambiguously.

R invests all the funds in sector 2 given $s_D = (0,0,0)$ if

$$p_0^D \underset{0}{\Delta u_R}(0,1) + p_1^D \underset{-}{\Delta u_R}(1,1) + p_2^D \underset{-}{\Delta u_R}(2,1) + p_3^D \underset{-}{\Delta u_R}(3,1) < 0 \tag{64}$$

$$p_0^D \underset{-}{\Delta u_R}(1,1) + p_1^D \underset{-}{\Delta u_R}(2,1) + p_2^D \underset{-}{\Delta u_R}(3,1) + p_3^D \underset{-}{\Delta u_R}(4,1) < 0 \tag{65}$$

$$p_0^D \underset{+}{\Delta u_R}(0,2) + p_1^D \underset{0}{\Delta u_R}(1,2) + p_2^D \underset{-}{\Delta u_R}(2,2) + p_3^D \underset{-}{\Delta u_R}(3,2) < 0 \tag{66}$$

$$p_0^D \underset{-}{\Delta u_R}(2,1) + p_1^D \underset{-}{\Delta u_R}(3,1) + p_2^D \underset{-}{\Delta u_R}(4,1) + p_3^D \underset{-}{\Delta u_R}(5,1) < 0 \tag{67}$$

$$p_0^D \underset{0}{\Delta u_R}(1,2) + p_1^D \underset{-}{\Delta u_R}(2,2) + p_2^D \underset{-}{\Delta u_R}(3,2) + p_3^D \underset{-}{\Delta u_R}(4,2) < 0 \tag{68}$$

$$p_0^D \underset{+}{\Delta u_R}(0,3) + p_1^D \underset{+}{\Delta u_R}(1,3) + p_2^D \underset{0}{\Delta u_R}(2,3) + p_3^D \underset{-}{\Delta u_R}(3,3) < 0 \tag{69}$$

Equations (64), (65), $(67)$and (68) hold unambiguously.

Therefore complete specialization is an equilibrium if (62), (66) and (69) are satisfied. Q.E.D.

Proof of Proposition 8.

$s_D = (0,0,0)$ is a best reply to $s_R = (1,2,2)$ if

$$p_0^R \underset{+}{\Delta u_D}(0,1) + p_1^R \underset{+}{\Delta u_D}(0,2) + p_2^R \underset{+}{\Delta u_D}(0,3) + p_3^R \underset{+}{\Delta u_D}(1,3) > 0 \tag{70}$$

$$p_0^R \underset{+}{\Delta u_D}(1,1) + p_1^R \underset{+}{\Delta u_D}(1,2) + p_2^R \underset{+}{\Delta u_D}(1,3) + p_3^R \underset{+}{\Delta u_D}(2,3) > 0 \tag{71}$$

$$p_0^R \underset{+}{\Delta u_D}(0,2) + p_1^R \underset{+}{\Delta u_D}(0,3) + p_2^R \underset{+}{\Delta u_D}(0,4) + p_3^R \underset{+}{\Delta u_D}(1,4) > 0 \tag{72}$$

$$p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{+}{\Delta u_D}(2,3) + p_3^R \underset{+}{\Delta u_D}(3,3) > 0 \tag{73}$$

$$p_0^R \underset{+}{\Delta u_D}(1,2) + p_1^R \underset{+}{\Delta u_D}(1,3) + p_2^R \underset{+}{\Delta u_D}(1,4) + p_3^R \underset{+}{\Delta u_D}(2,4) > 0 \tag{74}$$

Equations $(70) - (72)$ and (74) hold unambiguously.

$s_R = (1,2,2)$ is a best reply to $s_D = (0,0,0)$ if equations $(64) - (68)$ hold and

$$p_0^D \underset{+}{\Delta u_R}(0,3) + p_1^D \underset{+}{\Delta u_R}(1,3) + p_2^D \underset{0}{\Delta u_R}(2,3) + p_3^D \underset{-}{\Delta u_R}(3,3) > 0 \tag{75}$$

Equations $(64), (65), (67)$and (68) hold unambiguously.

$s_D^* = (0,0,0)$ and $s_R^* = (1,2,2)$ is a Bayesian equilibrium if (66), (73) and (75) are satisfied. Q.E.D.

Proof of Proposition 9.

$s_D = (0,0,0)$ is best response to $s_R = (1,1,2)$ if

$$p_0^R \underset{+}{\Delta u_D}(0,1) + p_1^R \underset{+}{\Delta u_D}(0,2) + p_2^R \underset{+}{\Delta u_D}(1,2) + p_3^R \underset{+}{\Delta u_D}(1,3) > 0 \tag{76}$$

$$p_0^R \underset{+}{\Delta u_D}(1,1) + p_1^R \underset{+}{\Delta u_D}(1,2) + p_2^R \underset{+}{\Delta u_D}(2,2) + p_3^R \underset{+}{\Delta u_D}(2,3) > 0 \tag{77}$$

$$p_0^R \underset{+}{\Delta u_D}(0,2) + p_1^R \underset{+}{\Delta u_D}(0,3) + p_2^R \underset{+}{\Delta u_D}(1,3) + p_3^R \underset{+}{\Delta u_D}(1,4) > 0 \tag{78}$$

$$p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{-}{\Delta u_D}(3,2) + p_3^R \underset{+}{\Delta u_D}(3,3) > 0 \tag{79}$$

$$p_0^R \underset{+}{\Delta u_D}(1,2) + p_1^R \underset{+}{\Delta u_D}(1,3) + p_2^R \underset{+}{\Delta u_D}(2,3) + p_3^R \underset{+}{\Delta u_D}(2,4) > 0 \tag{80}$$

Equations $(76) - (78)$and (80) hold unambiguously.

$s_R = (1,1,2)$ is best response to $s_D = (0,0,0)$ if equations $(64), (65), (67), (68)$ and (75) hold and

$$p_0^D \underset{+}{\Delta u_R}(0,2) + p_1^D \underset{0}{\Delta u_R}(1,2) + p_2^D \underset{-}{\Delta u_R}(2,2) + p_3^D \underset{-}{\Delta u_R}(3,2) > 0 \tag{81}$$

Equations $(64), (65), (67)$and (68) hold unambiguously.

Therefore $s_D^* = (0,0,0)$ and $s_R^* = (1,1,2)$ is a Bayesian equilibrium if $(75), (79)$ and (81) hold. Q.E.D.

Proof of Proposition 10.

$s_D = (0, 0, 1)$ is a best response to $s_R = (1, 2, 2)$ if $(70) - (72)$ and (74) hold (and they hold unambiguously) and

$$p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{+}{\Delta u_D}(2,3) + p_3^R \underset{+}{\Delta u_D}(3,3) < 0 \tag{82}$$

$s_R = (1, 2, 2)$ is a best response to $s_D = (0, 0, 1)$ if

$$p_0^D \underset{0}{\Delta u_R}(0,1) + p_1^D \underset{-}{\Delta u_R}(1,1) + p_2^D \underset{-}{\Delta u_R}(2,1) + p_3^D \underset{-}{\Delta u_R}(2,2) < 0 \tag{83}$$

$$p_0^D \underset{-}{\Delta u_R}(1,1) + p_1^D \underset{-}{\Delta u_R}(2,1) + p_2^D \underset{-}{\Delta u_R}(3,1) + p_3^D \underset{-}{\Delta u_R}(3,2) < 0 \tag{84}$$

$$p_0^D \underset{+}{\Delta u_R}(0,2) + p_1^D \underset{0}{\Delta u_R}(1,2) + p_2^D \underset{-}{\Delta u_R}(2,2) + p_3^D \underset{0}{\Delta u_R}(2,3) < 0 \tag{85}$$

$$p_0^D \underset{-}{\Delta u_R}(2,1) + p_1^D \underset{-}{\Delta u_R}(3,1) + p_2^D \underset{-}{\Delta u_R}(4,1) + p_3^D \underset{-}{\Delta u_R}(4,2) < 0 \tag{86}$$

$$p_0^D \underset{0}{\Delta u_R}(1,2) + p_1^D \underset{-}{\Delta u_R}(2,2) + p_2^D \underset{-}{\Delta u_R}(3,2) + p_3^D \underset{-}{\Delta u_R}(3,3) < 0 \tag{87}$$

$$p_0^D \underset{+}{\Delta u_R}(0,3) + p_1^D \underset{+}{\Delta u_R}(1,3) + p_2^D \underset{0}{\Delta u_R}(2,3) + p_3^D \underset{+}{\Delta u_R}(2,4) > 0 \tag{88}$$

Equations (83), (84) and (86)– (88) hold unambiguously.

Therefore $s_D^* = (0, 0, 1)$ and $s_R^* = (1, 2, 2)$ is a Bayesian equilibrium if (82) and (85) are satisfied. Q.E.D.

Proof of Proposition 11.

$s_D = (0, 0, 1)$ is best response to $s_R = (1, 1, 2)$ if equations $(76) - (78)$ and (80) hold (and they hold unambiguously) and

$$p_0^R \underset{-}{\Delta u_D(2,1)} + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{-}{\Delta u_D}(3,2) + p_3^R \underset{+}{\Delta u_D}(3,3) < 0 \tag{89}$$

$s_R = (1,1,2)$ is best response to $s_D = (0,0,1)$ if equations (83), (84) and $(86) - (88)$ hold (and they hold unambiguously) and

$$p_0^D \underset{+}{\Delta u_R(0,2)} + p_1^D \underset{0}{\Delta u_R}(1,2) + p_2^D \underset{-}{\Delta u_R}(2,2) + p_3^D \underset{0}{\Delta u_R}(2,3) > 0 \tag{90}$$

$s_D^* = (0,0,1)$ and $s_R^* = (1,1,2)$ is a Bayesian equilibrium if (89) and (90) are satisfied. Q.E.D.

6.2.2 Proofs for the opposite preferences

D and R maximally invest one project in the less preferred sector when $n_i = 3$. Therefore we need to check only one condition for each agent.

D invests all the funds in sector 1 if and only if

$$\begin{aligned} & p_0^R u_D(3,0) + p_1^R u_D(4-\alpha,\alpha) + p_2^R u_D(5-\beta,\beta) + p_3^R u_D(6-\gamma,\gamma) \\ > \; & p_0^R u_D(2,1) + p_1^R u_D(3-\alpha,1+\alpha) + p_2^R u_D(4-\beta,1+\beta) + p_3^R u_D(5-\gamma,1+\gamma) \end{aligned}$$

$\Leftrightarrow$

$$p_0^R \Delta u_D(2,1) + p_1^R \Delta u_D(3-\alpha,1+\alpha) + p_2^R \Delta u_D(4-\beta,1+\beta) + p_3^R \Delta u_D(5-\gamma,1+\gamma) > 0 \tag{91}$$

R invests all the funds in sector 2 if and only if

$$\begin{aligned} & p_0^D u_R(0,3) + p_1^D u_R(1-\alpha,3+\alpha) + p_2^D u_R(2-\beta,3+\beta) + p_3^D u_R(3-\gamma,3+\gamma) \\ > \; & p_0^D u_R(1,2) + p_1^D u_R(2-\alpha,2+\alpha) + p_2^D u_R(3-\beta,2+\beta) + p_3^D u_R(4-\gamma,2+\gamma) \end{aligned}$$

$\Leftrightarrow$

$$p_0^D \Delta u_R(0,3) + p_1^D \Delta u_R(1-\alpha, 3+\alpha) + p_2^D \Delta u_R(2-\beta, 3+\beta) + p_3^D \Delta u_R(3-\gamma, 3+\gamma) < 0 \tag{92}$$

Proof of Proposition 12.

Using equations (91) and (92) we can prove that $s_D^* = (0,0,0)$ and $s_R^* = (1,2,3)$ is a Bayesian equilibrium if and only if:

$$p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{+}{\Delta u_D}(2,3) + p_3^R \underset{+}{\Delta u_D}(2,4) > 0$$

$$\Leftrightarrow p_0^R < -\frac{p_1^R \Delta u_D(2,2) + p_2^R \Delta u_D(2,3) + p_3^R \Delta u_D(2,4)}{\Delta u_D(2,1)} \equiv \widetilde{p}_0^R$$

and

$$p_0^D \underset{+}{\Delta u_R}(0,3) + p_1^D \underset{-}{\Delta u_R}(1,3) + p_2^D \underset{-}{\Delta u_R}(2,3) + p_3^D \underset{-}{\Delta u_R}(3,3) < 0$$

$$\Leftrightarrow p_0^D < -\frac{p_1^D \Delta u_R(1,3) + p_2^D \Delta u_R(2,3) + p_3^D \Delta u_R(3,3)}{\Delta u_R(0,3)} \equiv \widetilde{p}_0^D$$

Q.E.D.

Proof of Proposition 13.

$s_D^* = (0,0,1)$ and $s_R^* = (1,2,3)$ is a Bayesian equilibrium if and only if:

$$p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}(2,2) + p_2^R \underset{+}{\Delta u_D}(2,3) + p_3^R \underset{+}{\Delta u_D}(2,4) < 0$$

$$\Leftrightarrow p_0^R > \widetilde{p}_0^R$$

and

$$p_0^D \underset{+}{\Delta u_R}(0,3) + p_1^D \underset{-}{\Delta u_R}(1,3) + p_2^D \underset{-}{\Delta u_R}(2,3) + p_3^D \underset{-}{\Delta u_R}(2,4) < 0$$

$$\Leftrightarrow p_0^D < -\frac{p_1^D \Delta u_R\,(1,3) + p_2^D \Delta u_R\,(2,3) + p_3^D \Delta u_R\,(2,4)}{\Delta u_R(0,3)} \equiv \widehat{p}_0^D$$

$s_D^* = (0,0,0)$ and $s_R^* = (1,2,2)$ is a Bayesian equilibrium if and only if:

$$p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}\,(2,2) + p_2^R \underset{+}{\Delta u_D}\,(2,3) + p_3^R \underset{+}{\Delta u_D}\,(3,3) > 0$$

$$\Leftrightarrow p_0^R < -\frac{p_1^R \Delta u_D\,(2,2) + p_2^R \Delta u_D\,(2,3) + p_3^R \Delta u_D\,(3,3)}{\Delta u_D(2,1)} \equiv \widehat{p}_0^R$$

and

$$p_0^D \underset{+}{\Delta u_R}(0,3) + p_1^D \underset{-}{\Delta u_R}\,(1,3) + p_2^D \underset{-}{\Delta u_R}\,(2,3) + p_3^D \underset{-}{\Delta u_R}\,(3,3) > 0$$

$$\Leftrightarrow p_0^D > \widetilde{p}_0^D$$

Q.E.D.

Proof of Proposition 14.
$s_D^* = (0,0,1)$ and $s_R^* = (1,2,2)$ is a Bayesian equilibrium if and only if:

$$p_0^R \underset{-}{\Delta u_D}(2,1) + p_1^R \underset{+}{\Delta u_D}\,(2,2) + p_2^R \underset{+}{\Delta u_D}\,(2,3) + p_3^R \underset{+}{\Delta u_D}\,(3,3) < 0$$

$$\Leftrightarrow p_0^R > \widehat{p}_0^R$$

and

$$p_0^D \underset{+}{\Delta u_R}(0,3) + p_1^D \underset{-}{\Delta u_R}\,(1,3) + p_2^D \underset{-}{\Delta u_R}\,(2,3) + p_3^D \underset{-}{\Delta u_R}\,(2,4) > 0$$

$$\Leftrightarrow p_0^D > \widehat{p}_0^D$$

Q.E.D.

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[1] The word "processed" describes informally reproduced works that may not be commonly available through library systems.

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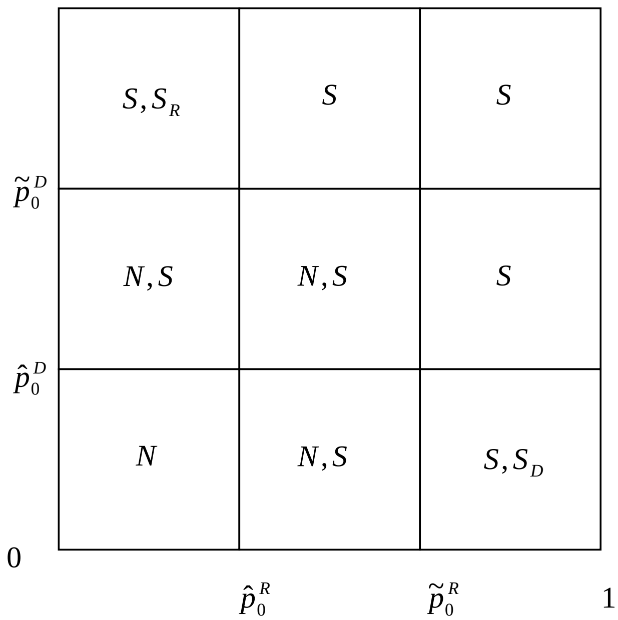
Figure 1.



$N : s_D = (0,0,0), s_R = (1,2,3)$
$S : s_D = (0,0,1), s_R = (1,2,2)$
$S_D : s_D = (0,0,1), s_R = (1,2,3)$
$S_R : s_D = (0,0,0), s_R = (1,2,2)$